Magnetic field of an isolated uniform sheet of current

Surface current density \( \vec{K} : \text{A/m} \)
Cut out a strip of width \( w \): \( I = Kw \)

Can think of this as parallel wires, direction \( \text{d-hat} \)
\( n \) wires/length, each carrying current \( K/n \), in the limit that \( n \)
goes to infinity

Symmetry tells us that the field is parallel to the sheet and perpendicular to the current, also that it is equal and opposite above and below the sheet

From Ampere’s law, get \( B \) above and below the sheet
Magnetic field of an infinite solenoid

Can think of this as stacked rings
n rings/length, each carrying current $I(n)$
Take the limit $n$ goes to infinity, $nI(n)$ stays constant

Symmetry tells us that the field is parallel to the axis
From Ampere’s law, get $B$ uniform inside, uniform outside

Fig. 29-19 Application of Ampere’s law to a section of a long ideal solenoid carrying a current $i$. The Amperian loop is the rectangle $abcd$. 
Magnetic field of an toroid

Can think of this as N stacked rings

Symmetry tells us that the field is tangent to the circle and the magnitude depends only on r

Ampere’s law $B(r) = \mu_0 NI/(2\pi r)$

![Diagram of a toroid with magnetic field lines showing the symmetry and field orientation.](image-url)
We’ve been talking about the magnetic fields produced in systems with steady currents
Biot-Savart law and Ampere’s law

Now back to electric fields
If I have a system with fixed charge distribution, I know how to find the E-field
Coulomb law and Gauss’ law
Described by potential which depends on the charge dist
Electric field is conservative – work is path independent, zero around closed path
Field lines cannot form closed loops

If I add steady currents to the system of fixed charges, the electric fields don’t change
Electric field from charge, magnetic field from current
Clicker: **True or false?** A fixed charge distribution can produce the electric field shown in the figure.

(a) true  
(b) false  
(c) not enough information to determine  
(d) I have no idea how to decide
Clicker: **True or false?** A fixed charge distribution can produce the electric field shown in the figure.

(a) true
(b) False – electric field produced by fixed charge distribution is conservative – field lines cannot form closed loops
(c) not enough information to determine
(d) I have no idea how to decide
Electric fields and \textbf{fixed charge distribution}

\begin{align*}
\mathbf{dE} &= k \frac{dq}{r^2} \hat{r} \\
\int_{S} \mathbf{E} \cdot d\mathbf{A} &= \frac{q_{\text{enc}}}{\varepsilon_0}
\end{align*}

Magnetic fields and \textbf{steady currents}

\begin{align*}
\mathbf{dB} &= \frac{\mu_0}{4\pi} \frac{i \, ds \times \hat{r}}{r^2} \\
\int_{C} \mathbf{B} \cdot d\mathbf{s} &= \mu_0 i_{\text{enc}}
\end{align*}

What happens when charges and currents change with time?
The DEMO

How can I get a current to flow in this coil?
Insert a battery into the loop
Potential difference across the ends –
Electric field along the wire

[Diagram of a coil with an arrow indicating the direction of current and a battery symbol]
The DEMO

How can I get a current to flow in this coil?

Current induced in a loop by a moving bar magnet

Rate of motion
Relative motion
N/S towards/away
The DEMO

How can I get a current to flow in this coil?

Current induced in a loop by a moving bar magnet

Rate of motion
Relative motion
N/S towards/away

**Changing** magnetic field
Verify: current induced in a loop by changing current in nearby loop
Electric field lines form a closed loop

THIS NEVER HAPPENS IN ELECTROSTATICS!

Changing magnetic field creates an electric field in addition to and DIFFERENT IN CHARACTER from that created by electric charges
New law that shows how CHANGES in magnetic field create this new kind of electric field

FARADAY’S LAW

\[ \int_C \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \]

- line integral
- surface integral (magnetic flux)

Loop C is the boundary of the surface S
Magnetic flux through a surface

Uniform magnetic field $\vec{B}$

Planar loop – flat surface $A, \hat{n}$

Flux $= (B \cos \theta) A$

Units $1 \text{T m}^2 = 1 \text{ Wb (weber)}$

\[
\Phi_B = \int \vec{B} \cdot d\vec{A}
\]
\( B(t) \) out of page increasing with time

\[
\Phi = B(t)A \\
d\Phi/dt \text{ is not zero}
\]

Area 0.2 m\(^2\)

\[
B(t) = (0.1\text{T/s}) \cdot t \\
d\Phi/dt = 0.02 \text{T m}^2/\text{s}
\]

Remember 1 T = 1 Ns/(Cm)

\[
d\Phi/dt = 0.02 \text{ Nm/C} = 0.02 \text{ V}
\]
The boundary of the surface $S$ is the closed loop $C$

$$\int_C \mathbf{E} \cdot d\mathbf{s}$$

Direction for line integral – if you walk along $C$ with normal vector to surface as up, you should go the direction that keeps the surface on your left

counterclockwise