Physics 271
Midterm 2
Date: November 13, 2017

There are 4 problems. Do all of them. The first two are short answer questions. Write just a couple of lines for the answer. The last two are require some solving. Show all your work. Cross things out neatly, DO NOT ERASE.

This is a closed textbook exam. You are allowed one sheet of 8.5” × 11” paper with your notes. Write your name on each blue-book you fill.

You need not solve the problems in the order given, but please indicate clearly which one you are solving. Please write neatly, and explain your work. Legibility is a critical factor in determining credit.
1. **SHORT ANSWER:** A rocket travels in a straight line in outer space (no external forces). If it starts at \( t = 0 \) with total mass \( M_0 \) and egests fuel at a time-dependent rate \( \gamma e^{-t/\tau} \) (where \( \tau \) is a constant) out the back with relative speed \( v_{ex} \), how fast will it be going at time \( t = \tau \)?
2. **SHORT ANSWER:** A bead of mass $m$ slides without friction on a rod that is made to rotate at a constant angular speed $\omega$. Neglect gravity. Assuming $r(t) = r_0 e^{\omega t}$, what is the tangential force exerted on the bead by the rod as a function of time?
3. An object consists of a uniform disk of mass $m$ and radius $R$ has a small particle of mass $m$ embedded at a point on the rim, as shown in the figure.

(a) Where is the center of mass of the object?
(b) What is the moment of inertia of the object around the center of mass?
(c) Make a sketch of the object resting on its rim on a flat surface (no tendency to rotate or translate). Be sure to show the embedded particle.
(d) The object rolls without slipping so that the center of the disk moves to the right at speed $v$. What are $x_{CM}(t)$ and $y_{CM}(t)$? Take $x = 0$ at the equilibrium position of the disk.
(e) If the center of the disk moves to the right at speed $v$, what is the angular velocity of the object around the center of mass? (Hint: draw the vector from the center of mass to the center of the disk and look at the angle it makes with the vertical)
(f) If the center of the disk moves to the right at speed $v$, what is the kinetic energy of the disk?
(g) If the object is rolled so that the center of the disk is displaced by a distance $x$ (small compared to $R$) and released without any initial translational or angular velocity, what is the frequency of small oscillations around equilibrium?
(h) If the object starts at its equilibrium point, what must the initial velocity of the center of the disk be for it not to return to equilibrium but to continue rolling away?
4. An asymmetric dumbbell consists of two point masses $M$ and $M/2$ connected by a massless rigid rod of length $2d$. The dumbbell is rotating freely around its center of mass on a flat frictionless table with angular velocity $\omega_0$. Initially the center of mass of the dumbbell is stationary; its translational velocity is zero.

A third mass $M/2$ is moving along a straight line towards the rotating dumbbell with linear velocity $v_0$. To simplify the algebra we assume that $v_0 = \omega_0 d$. When the third mass is very close to the dumbbell they collide such that the two masses $M/2$ form one mass $M$; the collision is completely inelastic and the dumbbell becomes symmetric. At the moment of collision the asymmetric dumbbell was at an angle $-180^\circ < \alpha < 180^\circ$ with respect to the direction of the velocity of the incoming mass. Ignore the fact that for some angles near to $180^\circ$ the incoming mass may hit the larger mass first.

(a) Find the translational velocity $v_1$ of the center of mass of the system after the collision as a function of the initial velocity $v_0$.

(b) Find the angular momentum of the dumbbell with respect to the midpoint of the rod before the collision.

(c) Find the angular velocity of the symmetric dumbbell after the collision, $\omega_1$, as a function of $\omega_0$ and $\alpha$.

(d) Find the change in the total kinetic energy of the three-mass system during the collision as a function of the given parameter. At what values of angle $\alpha$ is the loss of mechanical energy smallest?