Physics 271  Homework 1

1. \( \vec{A} = 2 \hat{i} - 3 \hat{j} + 7 \hat{k} \)
   \( \vec{B} = 5 \hat{i} + 1 \hat{j} + 2 \hat{k} \)

   Given two vectors \( \vec{A} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k} \)
   \( \vec{B} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k} \)

   a) \( \vec{A} + \vec{B} = (a_1 + b_1) \hat{i} + (a_2 + b_2) \hat{j} + (a_3 + b_3) \hat{k} \)
   \( \vec{A} \cdot \vec{B} = a_1 b_1 + a_2 b_2 + a_3 b_3 \)

   \( \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2) \hat{i} \\
   - (a_1 b_3 - a_3 b_1) \hat{j} \\
   + (a_1 b_2 - a_2 b_1) \hat{k} \)

   So:
   \( \vec{A} + \vec{B} = 7 \hat{i} - 2 \hat{j} + 9 \hat{k} \)
   \( \vec{A} - \vec{B} = -3 \hat{i} - 4 \hat{j} + 5 \hat{k} \)
   \( \vec{A} \cdot \vec{B} = (2 \times 9) + (-3 \times 1) + (7 \times 2) = 21 \)
   \( \vec{A} \times \vec{B} = -13 \hat{i} + 31 \hat{j} + 17 \hat{k} \)
b) The angles between two vectors can be found by using the dot product

\[ \vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos \theta_{AB} \]

\[ \theta_{AB} = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|} \right) \]

The z-axis base vector is \( \vec{k} \) (where magnitude is 1)

\[ \theta_A = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{k}}{|\vec{A}| \cdot |\vec{k}|} \right) \]

\[ \theta_A = \cos^{-1} \left( \frac{7}{\sqrt{4+9+49}} \right) = 0.476 \text{ rad} = 27.25^\circ \]

\[ \theta_B = \cos^{-1} \left( \frac{2}{\sqrt{25+1+4}} \right) = 1.20 \text{ rad} = 68.58^\circ \]

\[ \theta_{A \times B} = \cos^{-1} \left( \frac{17}{\sqrt{13^2 + 31^2 + 17^2}} \right) = 1.10 \text{ rad} = 63.17^\circ \]
First let us find $y$ as a function of $X', Y', \Theta$.

If you travel to the point $(X, Y)$ starting from the $X$-axis you reach the height $h = Y$ (obviously).

If instead you travel to the same point along the $X'$-axis and then along the $Y'$-axis you get $h = h_1 + h_2 = X' \sin \Theta + Y' \cos \Theta$.

So you get $Y = X' \sin \Theta + Y' \cos \Theta$.

Similarly (draw the axes & triangle for this yourself) you get $X = X' \cos \Theta - Y' \sin \Theta$.

**Hint:** Does this make sense for $\Theta = 0$ & $\Theta = \frac{\pi}{2}$?

We can write this as $\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} X' \\ Y' \end{pmatrix}$.
Initial acceleration:
\[ \int_0^t a_i \, dt = U_i(t) \] where \( a_i \) is a constant

and \[ \int_0^{t_1} U_i(t) \, dt = x_i = 25 \text{ m} \] where \( U_i \) is time dependent, \( t_i = 5 \text{ sec} \)

\[ U_i(0) = a_i \cdot 0 + \text{const} \] But since \( U_i(t) \) at \( t = 0 \) is 0
the bus started from rest
we get \( U_i(t) = a_i t \)

so we get \[ \int_0^t a_i t \, dt = x_i = \frac{1}{2} a_i t^2 \left|_0^5 \right. = \frac{1}{2} a_i \cdot 25 - 25 \text{ m} \]

a) \[ a_i = 2 \text{ m/s}^2 \]

b) \[ U_i(t = 5 \text{ sec}) = a_i (5) = \frac{10 \text{ m/s}}{5} = U_i(t = 5 \text{ sec}) \]

c) Braking acceleration:
\[ \Delta x_3 = U_i(\Delta t_3) + \frac{1}{2} a_3 (\Delta t_3)^2 = 18 \text{ m} \]

and \( 0 = U_i + a_3 \Delta t_3 \) (the final velocity is 0)

Solve for: \( \Delta t_3 = -\frac{U_i}{a_3} \) we know \( a_3 \) is negative
Plugging in to previous equation

\[
\Delta x_3 = v_i \left( -\frac{v_i}{a_3} \right) + \frac{1}{2} a_3 \left( -\frac{v_i}{a_3} \right)^2
\]

\[
\Delta x_3 = -\frac{1}{2} \frac{v_i^2}{a_3}
\]

Plug in 10 m/s = \(v_i\)  \(\Rightarrow\) \(a_3 = -2.8 \text{ m/s}^2\)

\(\Delta t_3 = -\frac{v_i}{a_3} = 3.6 \text{ s}\)

c) \(x_{\text{TOTAL}} = x_0 + \Delta x_2 + \Delta x_3 = 25 + (10,15) + 18 \)

\(x_{\text{TOTAL}} = 143 \text{ m}\)
4. We choose \( t = 0 \) \& \( z = 0 \) when the particle passes height \( A \) the first time.

Since acceleration is in the downward direction (and constant) we have

\[
\ddot{z} = \frac{dz}{dt^2} = -g
\]

Integrating we get:

\[
z(t) = z_0 + v_0 t - \frac{g}{2} \frac{t^2}{2}
\]

Where \( z_0 \& v_0 \) are the usual constants of integration.

From the graph it is clear that the time needed to reach point \( B \) from point \( A \)

\[
T_{AB} = \frac{T_A - T_B}{2}
\]

\[
h = z_B - z_A = v_0 T_B - \frac{gT_{AB}^2}{2}
\]
We have two unknowns \((g, v)\) and one equation. We need to get rid of \(v\) (the starting velocity).

To do this, note that \(z(t=0) = z(t=T_A)\)

\[ h = \frac{g T_A}{2} (T_A - T_B) - \frac{g}{2} \left(\frac{T_A - T_B}{2}\right)^2 \]

Plugging in for \(T_{AB}\)

\[ h = \frac{g T_A}{2} \left(\frac{T_A}{2} - \frac{T_B}{2}\right) - \frac{g}{2} \left(\frac{T_A - T_B}{2}\right)^2 \]

note that the \(T_A T_B\) term disappears.

\[ g = \frac{8h}{\frac{T_A^2}{2} - \frac{T_B^2}{2}} \]