

271 - Honors Physics 1
Final Exam Solutions

A particle moves in a plane so that its coordinates are

$$y(t) = (1 \text{ m}) + (3\text{m/s}^2) \cdot t^2 + 2\text{m} \cdot \sin((3\text{s}^{-1}) \cdot t)$$
$$x(t) = (-3 \text{ m}) + (2\text{m/s}^3) \cdot t^3$$

1A. (5pts) What is the speed of the particle at $t = 4 \text{ s}$?

$$v_x = x'(t) = 6 \times t^2$$
$$v_y = y'(t) = 6 \times t + 2 \times 3 \times \cos(3t)$$

substitute numbers (remember, the “natural” units of angle are radians!) get
 $v = 100.3 \text{ m/s}$

1B. (5pts) What is the acceleration of the particle at $t = 6 \text{ s}$?

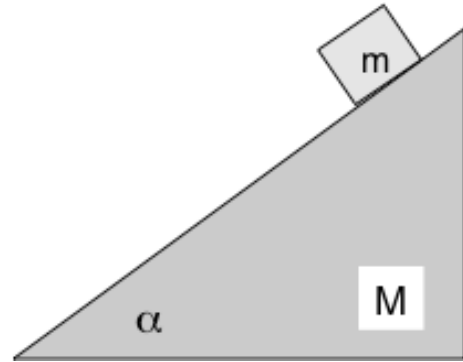
$$a_x = x''(t) = 12 \times t$$
$$a_y = y''(t) = 6 - 18 \times \sin(3t)$$

substitute numbers (remember, the “natural” units of angle are radians!) get
 $a = 74.6 \text{ m/s}^2$

A block of mass m is put on a triangular block of mass M and slope angle α as shown in the figure.

The surface between the small block and the triangular block is frictionless.

However, there is friction between the triangular block and the floor.



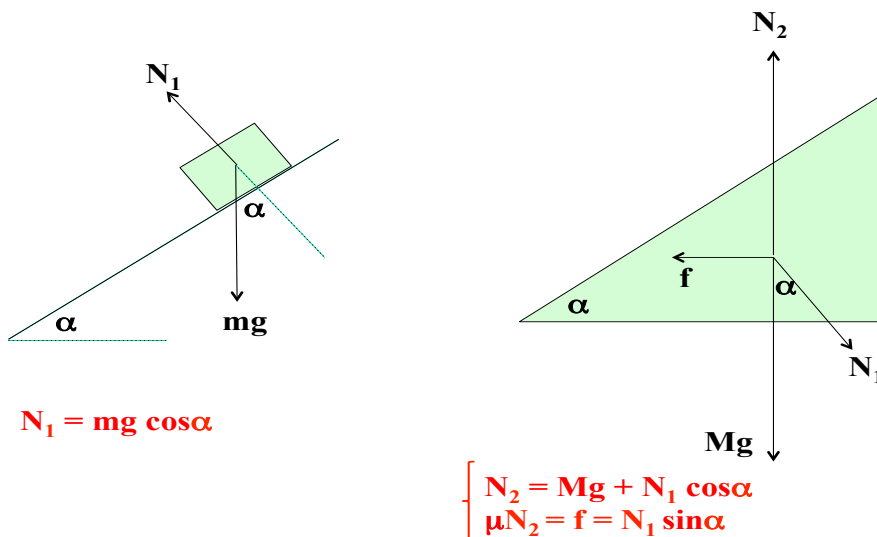
At $t=0$ both blocks are at rest, and the small block starts sliding down. The triangular block does not move, held in place by friction.

2. (10 pts) What is the minimum possible static friction coefficient μ_s between the triangular block and the floor?
(hint: be very careful when drawing the force diagram, and pay special attention to the Newton's Third law!)

The problem is really about the triangular block. It has four forces applied to it:

- its weight
- friction against the floor
- normal force from the floor
- force from the small rectangular block that is sliding off

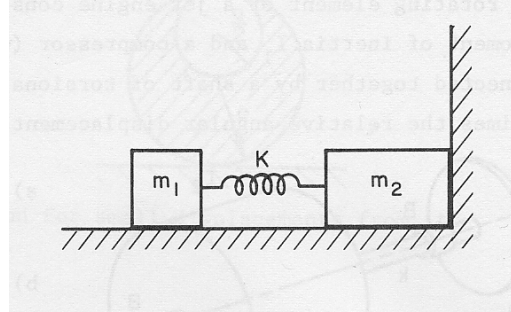
We can get the latter from the third Newton's law – it's the opposite in direction and same in magnitude as the normal force on the rectangular sliding block:



Solving the system of three equations (μ , N_1 and N_2 are unknowns) we can get

$$\mu = \frac{m \cdot \cos \alpha \cdot \sin \alpha}{M + m \cdot (\cos \alpha)^2}$$

Two unequal masses ($m_1 = m$; $m_2 = 2m$) rest on a frictionless table. They are connected by a spring of spring constant k . The spring is compressed a distance d , with m_2 resting against the wall. Then the system is released from rest.



3A. (3pts) How far does mass m_1 move before m_2 starts moving?

When the spring is compressed, the block m_2 is pushed against the wall, and the wall pushes back. Once block m_1 travels distance d , the spring begins to stretch and block m_2 will start moving (spring is then the only horizontal force)

After m_2 leaves the wall, motion consists of a translation motion of the center of mass and a vibrational motion about the center of mass

3B. (4 pts) What is the velocity of the center of mass?

Once block m_1 travels distance d , the system is isolated \rightarrow momentum conserves. At that moment the spring is uncompressed, m_2 is still at rest, but m_1 is moving with speed v_0 such that $\frac{1}{2} kd^2 = \frac{1}{2} mv_0^2$, so $v_0 = d \sqrt{k/m_1}$. From the momentum conservation we can write $(m_1 + m_2) v_{CM} = m_1 v_0$, so $v_{CM} = v_0 * m_1 / (m_1 + m_2) = 1/3 * d \sqrt{k/m_1}$

3C. (3 pts) What is the amplitude of the vibration? In other words, what is the maximum amount that the spring stretches for any time during the motion.

When the spring is maximally stretched or compressed, the two blocks would have the same speed, which, from momentum conservation again, would be v_{CM} . Therefore we can write:

$$\frac{1}{2} kd^2 = \frac{1}{2} kA^2 + \frac{1}{2} (m_1 + m_2) v_{CM}^2$$

substituting v_{CM} from above, we derive $A = d \sqrt{\frac{m_2}{m_1 + m_2}}$

let's check limit cases:

- if $m_1 \ll m_2$ then the block 1 is not really heavy enough to move block 2, so it's like block 2 is a part of the wall. Then the amplitude should be d , just as our equation says.

- if $m_1 \gg m_2$ then block 1 moves as if there's no block 2, and the amplitude should be almost zero, just as our equation says.

- for our case $m_2 = 2m_1$, we have $A = d \sqrt{2/3}$

A monatomic ideal gas is initially at a pressure of **1 atm (1 x 10⁵ Pa)** and a volume of **1 liter (0.001 m³)**.

It is expanded at constant pressure until the volume doubles.

4A. (5pts) How much work is done by the gas? (2 points)

$$W = P \Delta V = 1e5 \text{ Pa} * (2e-3 - 1e-3) \text{ m}^3 = 100 \text{ J}$$

4B. (5 pts) How much heat is put into the gas? (2 points)

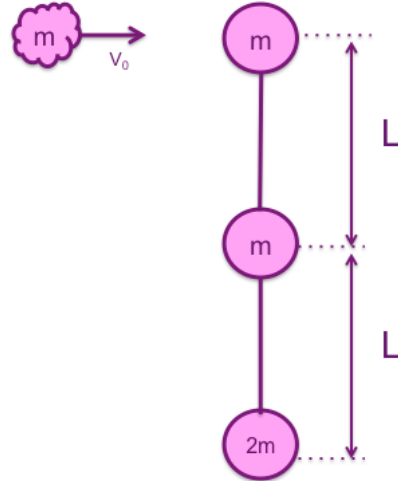
$$\Delta Q = \Delta E + W$$

$$\Delta E = 3/2 n R \Delta T = 3/2 (P_2 V_2 - P_1 V_1) = 3/2 P \Delta V$$

therefore,

$$\Delta Q = 5/2 P \Delta V = 250 \text{ J}$$

On a smooth table there are three small pucks of masses m , m , and $2m$ respectively, connected by light rigid rods of length L . A putty ball moving with speed V_0 strikes one of the pucks as shown in the figure and sticks to it. The system of pucks and putty then slides, rotating, on the table.



5A. (3pts) What is the speed of center of mass of the system of pucks and putty after the collision?

Momentum conservation: $mv_0 = 5mv_{CM}$,
 so $v_{CM} = \frac{1}{5}v_0$

5B. (4 pts) What is the angular rotation speed of the system?

Angular momentum conservation with respect to CM of putty+pucks:
 $mv_0L = I\omega = 4mL^2\omega$, so $\omega = \frac{v_0}{4L}$

5C. (3 pts) What is the speed of the puck with mass $2m$ right after the collision?

One can get this either from v_{CM} and ω , $v_2 = v_{CM} - L\omega = -v_0/20$

Or one can write angular momentum conservation w.r.t. putty after collision:

$$0 = m \cdot L \cdot v_{CM} - 2m \cdot 2L \cdot v_2$$

$$v_2 = -\frac{v_{CM}}{4} = -v_0/20$$

TOT

A satellite of mass m circles a planet of mass M in a circular orbit of radius R . It is then maneuvered into a circular orbit of radius $3R$.

6A. (5 pts) What minimum energy is required to make such maneuver?

For a satellite orbiting at radius R :

$$m \frac{v^2}{R} = G \frac{Mm}{R^2}$$

kinetic energy of satellite:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}G \frac{Mm}{R}$$

potential energy of the satellite

$$U = -G \frac{Mm}{R}$$

The total energy, therefore is

$$E_{TOT} = -\frac{1}{2}G \frac{Mm}{R}$$

So, the minimum energy required is $E_{TOT \text{ final}} - E_{TOT \text{ initial}} = E_{min} = \frac{1}{3}G \frac{Mm}{R}$

6B. (5 pts) How much would the kinetic energy of the satellite change?

From above, it follows that

$$\Delta K = -\frac{1}{3}G \frac{Mm}{R}$$