Problem 1.

A particle of mass $m$ slides without friction on the inside of a cone. The axis of the cone is vertical, and gravity is directed downward. The apex half-angle of the cone is $\theta$.

The path of the particle happens to be a circle in a horizontal plane. The speed of the particle is $v_0$.

Draw a force diagram and find the radius of the circular path in terms of $v_0$, $g$, and $\theta$.

Problem 2

A raindrop of initial mass $M_0$ starts falling from rest under the influence of gravity. Assume that the drop gains mass from the cloud at a rate proportional to the product of its instantaneous mass and its instantaneous speed: $dM/dt = kM|v|$, where $k$ is a constant. Neglect air resistance.

a) Using the Rocket Equation, find the differential equation relating the velocity of the raindrop to its acceleration.

b) Express $dv/dt$ from the equation you found in (a). It should show that as the speed increases, the acceleration $dv/dt$ decreases. At some speed, the acceleration should become zero. This means that the raindrop reached its constant terminal velocity.

c) Find the terminal velocity of the raindrop. Note, no need to solve the differential equation. Just use the arguments given in (b).

Problem 3

A block of mass $M$ starts sliding on a horizontal surface with the initial velocity $v_0$. The coefficient of friction between the block and the surface is $\mu$. How far will the block travel? How much time will it take for the block to stop moving? What is the total work done by the friction force in the process?

Problem 4.

A piece of uniform string of mass $M$ and length $L$ forms a circular loop. The loop is set spinning about the center of the circle with uniform angular velocity $\omega$. No gravity is present.

(a) Draw a force diagram for a small piece of the loop subtending a very small angle $d\theta$. What is the mass $dm$ of this piece in terms of $M$ and $d\theta$?

(b) Using the diagram you drew, find the tension of the string. Remember that $\sin(d\theta) = d\theta$ for small values of $d\theta$, expressed in radians.

Useful info: the length of the piece of the loop in (a) is $Rd\theta$, where $R$ is the radius of the loop, and $d\theta$ expressed in radians.