Physics 271 - Practice Midterm I - Solutions

1. At time $t = 0$ a car has a velocity $10\text{m/s}$ along the $x$ axis and it slows down with constant acceleration $-2\text{m/s}^2$. How long will it take for the car to stop?

(a) $10\text{s}$
(b) $5\text{s}$
(c) $20\text{s}$
(d) $2\text{s}$
(e) cannot be determined from the given data.

Constant acceleration model:

$$v = v_0 + a_x t = v_0 - 2t.$$ 

$v = 0 \Rightarrow t = v_0/2 = 5\text{s}$.

2. A car moving with velocity $24\text{m/s}$ along the $x$ axis starts slowing down with constant acceleration $-2\text{m/s}^2$. What is the total distance covered by the car before it stops?

(a) $9\text{m}$
(b) $12\text{ m}$
(c) $2\text{ m}$
(d) $144\text{ m}$
(e) $24\text{ m}$

Constant acceleration model:

$$v^2 = v_0^2 + 2a_x(x - x_0) = v_0^2 - 4(x - x_0)$$

$v = 0 \Rightarrow x - x_0 = 24^2/4\text{m} = 144\text{m}$. 

1
3. A particle moves counterclockwise with constant speed on a circular trajectory of radius \( R = \sqrt{2} \) m centered at the origin in the \( xy \)-plane. The velocity vector of the particle at some time \( t \) is \( \vec{v} = (1 \text{ m/s})(-\hat{i} + \hat{j}) \). What is the acceleration vector of the particle at the same time \( t \)?

(a) \( \vec{a} = 0 \)
(b) \( \vec{a} = (1 \text{ m/s}^2)(\hat{i} + \hat{j}) \)
(c) \( \vec{a} = (1 \text{ m/s}^2)(\hat{i} - \hat{j}) \)
(d) \( \vec{a} = -(1 \text{ m/s}^2)(\hat{i} + \hat{j}) \)
(e) \( \vec{a} = (0.7 \text{ m/s}^2)(-\hat{i} + \hat{j}) \)

In circular uniform motion the centripetal acceleration is always perpendicular to the velocity and points toward the center along the radial direction.

\[
|\vec{a}| = \frac{v^2}{R} = \frac{(-\hat{i} + \hat{j}) \cdot (-\hat{i} + \hat{j})}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\text{ m/s}^2.
\]

Then

\[
\vec{a} = -(1 \text{ m/s}^2)(\hat{i} + \hat{j}).
\]

since

\[
|\hat{i} + \hat{j}| = \sqrt{2}, \quad (\hat{i} + \hat{j}) \cdot (-\hat{i} + \hat{j}) = 0
\]

and \(-\hat{i} + \hat{j}\) points radially towards the center.

4. At \( t = 0 \) two particles start moving simultaneously from the point with coordinates \( x = -1, \ y = 0 \) in the \( xy \) plane. Particle \( A \) travels clockwise on a circular trajectory centered at the origin with constant speed \( v \). Particle \( B \) travels with constant velocity \( \vec{v} = \hat{v} \) along the \( x \) axis. What is the velocity \( \vec{v}_{AB} \) of particle \( A \) relative to \( B \) at time \( t = 0? \)

(a) \( \vec{v}_{AB} = v(\hat{j} - \hat{i}) \)
(b) \( \vec{v}_{AB} = \hat{v} \)
(c) \( \vec{v}_{AB} = v(\hat{i} + \hat{j}) \)
(d) \( \vec{v}_{AB} = 0 \)
\( (e) \vec{v}_{AB} = -v\hat{j}. \)

The initial velocity of particle A is \( \vec{v}_{A0} = v\hat{j} \) and the initial velocity of particle B is \( \vec{v}_{B0} = v\hat{i}. \) At time \( t = 0 \) we have

\[
\vec{v}_{A0} = \vec{v}_{B0} + \vec{v}_{AB}.
\]

Therefore

\[
\vec{v}_{AB} = \vec{v}_{A0} - \vec{v}_{B0} = v(\hat{j} - \hat{i}).
\]

5. Two projectiles are shot simultaneously with initial velocities \( \vec{v}_{0A} = (10 \text{m/s})\hat{i}, \vec{v}_{0B} = -(10 \text{m/s})\hat{i} \) in the vertical plane. The initial position vector of A is \( \vec{r}_{0A} = (78.4 \text{m})\hat{j} \) and the initial position vector of B is \( \vec{r}_{0B} = (80 \text{m/s})\hat{i} + (78.4 \text{m/s})\hat{j}. \) Which of the following will happen?

(a) The projectiles will collide in air before they hit the ground.

(b) The projectiles will collide precisely when they simultaneously hit the ground.

(c) The projectiles will miss each other in the air.

(d) Projectile A will hit the ground before projectile B.

(e) Projectile B will hit the ground before projectile A.

For this problem \( g = 9.8 \text{m/s}^2 \) and the ground is at \( y = 0. \)

Note that \( y_0 = 78.4 \text{m} \) for both projectiles. The \( y \)-components of the initial velocities are zero for both projectiles. Therefore both of them will reach the ground in a time interval

\[
\Delta t = \sqrt{\frac{2y_0}{g}} = 4 \text{s}
\]

The \( x \) coordinate of the first projectile at time \( t = 4 \text{s} \) is

\[
x_A = 4v_{0A} = 40 \text{m}.
\]
The $x$ coordinate of the second projectile at time $t = 4s$ is

$$x_B = 80 - 4v_0 = 80 - 40 = 40m.$$ 

Therefore they reach the ground at the same point and same time.

6. A projectile is shot from the origin with initial velocity $\vec{v}_0 = (20m/s)\hat{i} + (30m/s)\hat{j}$. A vertical obstacle of height $h$ is placed at the point with coordinates $x = 40m$ on the $x$-axis. What is the maximum value of $h$ for which the projectile will not hit the obstacle?

(a) 20m  
(b) 30m  
(c) 9.8m  
(d) 39.2m  
(e) 40.4m.

The $x$ and $y$ coordinates of the projectile are given by

$$x = v_{0x}t = 20t \quad y = v_{0y}t - gt^2/2 = 30t - 4.9t^2.$$ 

The projectile reaches $x = 40m$ in a time 

$$t = 40/20 = 2s.$$ 

The $y$ coordinate at time $t = 2s$ is 

$$y = 60 - 19.6 = 40.4m.$$ 

Therefore $h_{max} = 40.4m$.

7. A ball with mass $m$ is thrown in the air with initial speed $\vec{v}_0$ at an angle $\theta$ above the horizontal. In addition to gravity, the ball is subject to a force $\vec{F} = -\gamma \vec{v}$. What is the $y$-component of the acceleration vector of the ball at time $t = 0$?
\[(a) \ a_y = -g - (\gamma/m)\nu_0 \sin \theta \]
\[(b) \ a_y = -g \]
\[(c) \ a_y = -g + (\gamma/m)\nu_0 \sin \theta \]
\[(d) \ a_y = g + \gamma\nu_0 \cos \theta \]
\[(e) \ a_y = 0 \]

Newton’s second law:
\[\vec{F}_g + \vec{F} = m\vec{a} \]

**y-component:**
\[-mg - \gamma\nu_0 \sin \theta = ma_y \]
\[a_y = -g - (\gamma/m)\sin \theta. \]

**8.** Two astronauts of masses \(m_1 = 70\text{kg}\) and \(m_2 = 80\text{kg}\) pull in opposite directions from the two ends of an ideal cord stretching along the \(x\)-axis. The acceleration of the first astronaut is \(\vec{a}_1 = (1\text{m/s}^2)i\). Assuming there are no external forces acting on them, what is the acceleration of the second astronaut?

\[(a) \ \vec{a}_2 = (1\text{m/s}^2)i \]
\[(b) \ \vec{a}_2 = -(1\text{m/s}^2)i \]
\[(c) \ \vec{a}_2 = (8/7 \text{ m/s}^2)i \]
\[(d) \ \vec{a}_2 = -(7/8 \text{ m/s}^2)i \]
\[(e) \ \vec{a}_2 = (7/8 \text{ m/s}^2)i \]

Newton’s second law:
\[\vec{F}_1 = m_1\vec{a}_1 \quad \vec{F}_2 = m_2\vec{a}_2 \]

For ideal cord
\[\vec{F}_1 + \vec{F}_2 = 0. \]
Therefore
\[ m_1\ddot{a}_1 + m_2\ddot{a}_2 = 0. \]

9. In Problem 8, what is the relative acceleration of the second astronaut with respect to the first?

(a) \[ \ddot{a}_2 = (16/7 m/s^2)\hat{i} \]

(b) \[ \ddot{a}_2 = -(15/8 m/s^2)\hat{i} \]

(c) \[ \ddot{a}_2 = (15/8 m/s^2)\hat{i} \]

(d) \[ \ddot{a}_2 = -(15/7 m/s^2)\hat{i} \]

(e) \[ \ddot{a}_2 = 2 m/s^2\hat{i} \]

We have
\[ \ddot{a}_2 = \ddot{a}_1 + \ddot{a}_{21} \]
where \( \ddot{a}_{21} \) is the acceleration of the second astronaut relative to the first. Therefore
\[ \ddot{a}_{21} = \ddot{a}_2 - \ddot{a}_1 = -(15/8 m/s^2)\hat{i}. \]

10. Two identical balls of mass \( m \) each are tied together by an ideal cord of length \( l \). The balls are set in uniform circular motion around the midpoint with constant speed \( v \) such that the cord is stretched taut. What is the magnitude of tension in the cord?

(a) \( mv^2/l \)

(b) \( mv^2/2l \)

(c) \( 2mv^2/l \)

(d) \( mlv^2 \)

(e) cannot be determined from the data.

Newton’s second law:
\[ \vec{T} = m\ddot{a} \]
where \( \vec{a} \) is the centripetal acceleration. The magnitude of the centripetal acceleration is

\[
a = \frac{v^2}{r} = \frac{2v^2}{l}.
\]