Physics 271 - Practice Midterm II - Solutions

1. A stretched spring pulls a block of wood of mass \( m = 0.1 \) kg up a ramp of angle \( \theta = 30^\circ \). The kinetic friction coefficient between the block and the ramp is \( \mu_k = \frac{1}{\sqrt{3}} \) and the spring constant is \( k = 10 \) N/m. The elastic force from the spring is parallel to the ramp. Find the acceleration of the block when the spring is stretched a distance \( \Delta l = 0.1 \) m from its relaxed position.

(a) 0.2 m/s\(^2\)
(b) 0.4 m/s\(^2\)
(c) 1.6 m/s\(^2\)
(d) 2.1 m/s\(^2\)
(e) 0.8 m/s\(^2\)

For this problem note that
\[
\sin(30^\circ) = \frac{1}{2}, \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}.
\]

Solution. Choose the \( x \) axis pointing upward along the ramp and the \( y \) axis orthogonal to the ramp and pointing away from the ramp. Then Newton’s 2nd law for the block yields
\[
ma_x = k\Delta l - mgsin\theta - \mu_kN
\]
\[
ma_y = N - mgcos\theta = 0
\]
Therefore
\[
a_x = \frac{k\Delta l}{m} - g(sin\theta + \mu_kcos\theta) = 10 - 9.8 = 0.2 \text{ m/s}^2.
\]

2. A block of mass \( m = 0.1 \) kg is tied at the end of an ideal cord wrapped around an ideal pulley as shown below. The other end of the cord is tied to a fixed spring of constant \( k = 2 \) N/m. The spring is initially relaxed, the cord
is taut and the block is released with zero initial velocity. How far will the
block fall before it bounces back?

\[ (a) \ 1.23 \text{m} \]
\[ (b) \ 0.67 \text{m} \]
\[ (c) \ 0.49 \text{m} \]
\[ (d) \ 1.16 \text{m} \]
\[ (e) \ 0.98 \text{m} \]

**Solution.** There is no friction in the system, hence the mechanical energy
is conserved. Initially the block has gravitational potential energy \( mgh \).
When it bounces back it has only elastic potential energy \( kh^2/2 \). Therefore

\[ mgh = \frac{kh^2}{2} \Rightarrow h = \frac{2mg}{k} = 0.98 \text{m} \]

3. In problem 2, what is the speed of the block when it has dropped a
distance \( d = 0.49 \text{m} \)?

\[ (a) \ 0.59 \text{m/s} \]
\[ (b) \ 2.19 \text{m/s} \]
\[ (c) \ 2.87 \text{m/s} \]
\[ (d) \ 1.36 \text{m/s} \]
\[ (e) \ 2.49 \text{m/s} \]
Solution. Again, mechanical energy is conserved, hence

\[ mgd = \frac{kd^2}{2} + \frac{mv^2}{2} \]

This yields

\[ v = \sqrt{2gd - kd^2/m} = 2.19\text{m/s} \]

4. A ball hangs at the end of an ideal cord of length \( l = 1\text{m} \), and the other end of the cord is fixed. The ball is given a horizontal initial velocity of magnitude \( v \). What is the minimum value of \( v \) such that the ball moves on a circular trajectory of radius \( l \) in the vertical plane.

(a) 7m/s  
(b) 4m/s  
(c) 5.6m/s  
(d) 6m/s  
(e) 3.2m/s

Solution. Since there is no friction, the mechanical energy is conserved. Initially the ball has kinetic energy

\[ K_0 = \frac{mv^2}{2} \]

Suppose it moves on a circular trajectory. At the top it has both kinetic and potential gravitational energy

\[ K_{\text{top}} = 2mgl + \frac{mv_{\text{top}}^2}{2} \]

Newton’s 2nd law at the top yields

\[ ma_c = T + mg \]

where \( a_c = \frac{mv_{\text{top}}^2}{l} \) is the centripetal acceleration and \( T \) is the magnitude of the tension in the cord. Since \( T \geq 0 \), it follows that

\[ v_{\text{top}}^2 \geq gl. \]
The lowest initial speed is obtained from energy conservation:
\[
\frac{mv^2}{2} = 2mgl + \frac{mgl}{2} = \frac{5mgl}{2}
\]
Hence
\[
v = \sqrt{5gl} = 7\text{m/s}.
\]

5. A block of mass \(m = 0.2\text{kg}\) is tied at the end of a spring of constant \(k = 20\text{N/m}\) as shown below. The other end of the spring is held fixed. The spring is initially compressed a distance \(\Delta l = 0.1\text{m}\) and the surface of the table is rough. The block is given a horizontal initial velocity of magnitude \(v_0 = 4\text{m/s}\). The velocity of the block when the spring is relaxed is \(v_1 = 3\text{m/s}\). Find the work done by kinetic friction during this part of the motion.

\[
(a)\ 1.2\text{J} \\
(b)\ -0.6\text{J} \\
(c)\ -0.8\text{J} \\
(d)\ 0.4\text{J} \\
(e)\ 0\text{J}
\]

**Solution.** By conservation of total energy
\[
\Delta E_{\text{mec}} + \Delta E_{\text{th}} = 0
\]
where \(\Delta E_{\text{th}} = |W_{f_k}| = -W_{f_k}\). Therefore
\[
W_{f_k} = \Delta E_{\text{mec}} = \frac{mv_1^2}{2} - \frac{mv_0^2}{2} - \frac{k(\Delta l)^2}{2} = -0.8\text{J}
\]
6. In the set-up of problem 5 suppose the kinetic friction coefficient between the block and the horizontal surface is \( \mu_k = 0.2 \). What is the horizontal component of the acceleration of the block immediately after it starts moving?

(a) 4.16m/s\(^2\)
(b) 2.88m/s\(^2\)
(c) \(-2.56m/s^2\)
(d) \(-3.96m/s^2\)
(e) 1.74m/s\(^2\)

**Solution.**

\[
N = mg + k\Delta l = 3.96N
\]

\[
f_k = \mu_k N
\]

\[
a_x = \frac{-f_k}{m} = -3.96m/s^2
\]

7. A ball of mass \( m = 0.1kg \) is dropped from a height \( h = 11m \) above the surface of a triangular block of mass \( M = 1kg \) as shown below. The triangular block is at rest on a frictionless table and the kinetic energy is conserved during the collision. Suppose the ball moves horizontally immediately after it hits the block. Find the speed of the block after collision.

\[
(a) 0.5m/s
\]
\[
(b) 0.1m/s
\]
Solution. The velocity of the ball just before it hits the block is

\[ \vec{v}_0 = -\sqrt{2gh}\hat{j}. \]

Let \( \vec{v}_1 = v_1\hat{i} \) denote the velocity of the ball after collision, which is horizontal by assumption. Let \( \vec{u} = u_x\hat{i} \) denote the velocity of the block after collision. Since there is no external force along the x-axis, the x-component of the total momentum is conserved. Therefore

\[ Mu_x + mv_1 = 0 \]

Moreover, since the kinetic energy is conserved during the collision, we have

\[ Mu_x^2 + mv_1^2 = mv_0^2 \]

The above equations imply

\[ u = \sqrt{\frac{2m^2}{M(m+M)}gh} = 1.4 \text{m/s} \]

8. A triangular plate of angle \( \theta = 30^\circ \) is glued to the surface of a horizontal table. A ball moving with speed \( v_0 = 10 \text{ m/s} \) hits the plate sideways as shown below. The picture shows the collision seen from above such that the \((x,y)\) plane is horizontal. There is no friction between the ball and the plate and no friction between the ball and the table. Suppose the ball slides along the face of the plate after collision. Find the final speed of the ball.

For this problem note that

\[ \sin(30^\circ) = \frac{1}{2} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}. \]
(a) 10m/s  
(b) 7m/s  
(c) 12m/s  
(d) 0m/s  
(e) 5m/s  

_Solution._ Choose a coordinate system as follows.

The only force acting on the ball during the collision is the normal force \( \vec{N} \) which is orthogonal to the surface. Therefore the parallel component of the velocity must be preserved during the collision. This yields

\[ mv_\parallel = mv_0 \sin \theta \]

after the collision. Hence \( v_\parallel = v_0 \sin \theta = 5 \text{m/s} \).

9. In problem 8 suppose the mass of the ball is \( m = (\sqrt{3}/10) \text{kg} \). Compute
the magnitude of the impulse of the normal force acting on the ball during
the collision.

(a) 1 kg · m/s
(b) 1.5 kg · m/s
(c) 0.5 kg · m/s
(d) 0.7 kg · m/s
(e) 1.7 kg · m/s

Solution. The impulse of the normal force acting on the ball is given by

\[ \vec{J}_N = \Delta \vec{p}_\perp. \]

Since the ball slides along the surface after collision,

\[ \Delta \vec{p}_\perp = mv_0 \cos \theta = 1.5 \text{ kg} \cdot \text{m/s}. \]

Therefore

\[ J_N = 1.5 \text{ kg} \cdot \text{m/s}. \]

10. Suppose the kinetic energy is conserved during the collision in problem 8. Find the velocity vector \( \vec{v} \) of the ball after collision.

(a) \( \vec{v} = -(10\sqrt{3}/s)\hat{i} \)
(b) \( \vec{v} = -(5\text{m/s}) (\hat{i} - \sqrt{3}\hat{j}) \)
(c) \( \vec{v} = (5\text{m/s}) (\hat{i} + \sqrt{3}\hat{j}) \)
(d) \( \vec{v} = (10\text{m/s}) (\sqrt{3}\hat{i} - \hat{j}) \)
(e) \( \vec{v} = (5\text{m/s}) (\sqrt{3}\hat{i} + \hat{j}) \)

Solution. The parallel component of the momentum is still preserved
since there is no force on the ball along this direction. Hence after collision

\[ v_\parallel = v_0 \sin \theta = \frac{v_0}{2}. \]
The magnitude of the perpendicular component after collision follows from energy conservation

\[ v_\perp = \sqrt{v_0^2 - v_\parallel^2} = \frac{v_0 \sqrt{3}}{2} \]

Moreover, the perpendicular component will point away from the surface since the ball bounces back. Therefore the picture after collision is the following:

Then we have

\[ \vec{v}_\perp = \frac{v_0 \sqrt{3}}{2} (\sin \theta \hat{i} + \cos \theta \hat{j}) = \frac{v_0 \sqrt{3}}{4} (\hat{i} + \sqrt{3} \hat{j}) \]

and

\[ \vec{v}_\parallel = \frac{v_0}{2} (\cos \theta \hat{i} - \sin \theta \hat{j}) = \frac{v_0}{4} (\sqrt{3} \hat{i} - \hat{j}) \]

Therefore

\[ \vec{v} = \vec{v}_\perp + \vec{v}_\parallel = \frac{v_0}{2} (\sqrt{3} \hat{i} + \hat{j}) \]