Midterm 1: Monday October 10th 2015

- Motion in one, two and three dimensions
- Forces and Motion I (Newton’s laws)
- No energy and work.
- Practice problems and sample exam posted at physics.rutgers.edu/ugrad/271/exams.html

Webassign Troubleshooting

- If correct answers are marked incorrect by the system, please send me a screenshot of the problem in question and your answer after the due date.
5. Forces and Motion I

- **Force:** ~ physical effect which changes the velocity of an object

- **Newton’s 1st law**

  If no net force acts on a body,
  \[
  \vec{F}_{\text{net}} = 0,
  \]
  the body’s velocity cannot change;
  the body cannot accelerate.

- **Inertial System:** reference system where
  Newton’s laws are valid
What is the main effect of a nonzero net force?

- An applied force can set a static object in motion or can stop a moving object.

\[ \downarrow \]

change in velocity, hence acceleration.
What is the relation between the net force acting on an object and the resulting acceleration?

\[ \vec{F}_{\text{net}} = m\vec{a}, \]

- \( m \): Mass, an intrinsic characteristic of a body relating a force on the body to the resulting acceleration.

Newton’s second law
- **Force unit:** Newton (N)

![Image](image.jpg)

**1 Newton** is the force exerted on a standard mass of 1 kg to produce an acceleration of 1 m/s².
i-Clicker

The graph below represents the $x$-component of the velocity of an object as a function of time.

Which of the following graphs represents the time dependence of the $x$-component of the net force acting on the object?
Answer
The graph below represents the $x$-component of the velocity of an object as a function of time.

Which of the following graphs represents the time dependence of the $x$-component of the net force acting on the object?

A B C D
**Example:** puck on frictionless ice

A puck of mass $m$ is simultaneously pulled by two ideal cords as shown above. What is its acceleration? Free body diagram.

Recall: ideal cord = massless, unstretchable cord
Newton’s second law:

\[ \vec{F}_{\text{net}} = m\vec{a} \]

\[ \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 \]

\[
\begin{align*}
(\vec{F}_{\text{net}})_x &= F_{1x} + F_{2x} \\
&= F_1 \cos \alpha - F_2 \sin \beta \\
(\vec{F}_{\text{net}})_y &= F_{1y} + F_{2y} \\
&= F_1 \sin \alpha - F_2 \cos \beta
\end{align*}
\]

\[
\begin{align*}
ma_x &= F_1 \cos \alpha - F_2 \sin \beta \\
ma_y &= F_1 \sin \alpha - F_2 \cos \beta
\end{align*}
\]
• **Gravitational force**

  • Pull force $\vec{F}_g$ acting on any object near the Earth’s surface. How do we compute it?

  • Free fall acceleration $\vec{a} = -g\hat{j}$.

  • Newton’s second law for freely falling object:

    $$\vec{F}_g = -mg\hat{j}$$

  • **Note:** approximation of a more general theory (studied later)
• **Weight:** magnitude of gravitational force acting on the object.

\[ W = F_g = mg \]
• **Normal force:** $\vec{F}_N$ stops the object from falling through; always perpendicular to the surface of the surface.

The normal force is the force on the block from the supporting table.

The gravitational force on the block is due to Earth's downward pull.

\[ \vec{F}_N + \vec{F}_g = m\vec{a} \Rightarrow (F_N)_y + (F_g)_y = ma_y \Rightarrow F_N = m(g + a_y) \]

$a_y$ vertical acceleration of table + block
• **Example:** box pulled up a frictionless inclined plane by an ideal cord

\[
\vec{F}_{\text{net}} = m\vec{a} \quad \vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_N + \vec{T}
\]

\[
T - mg \sin \theta = ma_x \quad F_N - mg \cos \theta = ma_y
\]

\[
a_y = 0 \text{ (no motion along } y \text{ axis)} \Rightarrow F_N = mg \cos \theta.
\]

\[
a_x = (T - mg \sin \theta)/m
\]
i-Clicker

A box of mass $m$ is pulled along a frictionless table as shown below. What is the horizontal component $a_x$ of the acceleration of the box?

A) $a_x = 0$
B) $a_x = -(T/m) \cos \theta$
C) $a_x = T/m$
D) $a_x = (T/m) \sin \theta$
E) $a_x = -T/m$
Answer

A box of mass $m$ is pulled along a frictionless table as shown below. What is the horizontal component $a_x$ of the acceleration of the box?

\[ A) \ a_x = 0 \]
\[ B) \ a_x = -(T/m)\cos \theta \]
\[ C) \ a_x = T/m \]
\[ D) \ a_x = (T/m)\sin \theta \]
\[ E) \ a_x = -T/m \]
\[ \vec{F}_{\text{net}} = m\vec{a} \]
\[ \vec{F}_{\text{net}} = \vec{T} + \vec{F}_g + \vec{F}_N \]
\[ (F_{\text{net}})_x = T_x = -T\cos\theta \]
\[ ma_x = -T\cos\theta \]
\[ a = -(T/m)\cos\theta \]
A box of mass $m$ is pulled along a frictionless table as shown below. For which values of $T, m, \theta$ will the box remain on the table?

- A) For all values of $T, m, \theta$
- B) $T \sin \theta > mg$
- C) $T \sin \theta \leq mg$
- D) $T \cos \theta < mg$
- E) None one the above
Answer

A box of mass $m$ is pulled along a frictionless table as shown below. For which values of $T, m, \theta$ will the box remain on the table?

- **A)** For all values of $T, m, \theta$
- **B)** $T \sin \theta > mg$
- **C)** $T \sin \theta \leq mg$
- **D)** $T \cos \theta < mg$
- **E)** None one the above
The condition for the box to stay on the table is

\[ a_y = 0 \Rightarrow T \sin \theta + F_N - mg = 0. \]

Note that \( F_N \geq 0 \) since it is the magnitude of \( \vec{F}_N \).

Hence

\[ T \sin \theta \leq mg \]
Newton’s Third Law

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.
\[ \vec{F}_{BC} + \vec{F}_{CB} = 0 \]

\[ F_{BC} = F_{CB} \]

The force on $B$ due to $C$ has the same magnitude as the force on $C$ due to $B$. 
\[ \vec{F}_{CE} + \vec{F}_{EC} = 0 \]
\[ \vec{F}_{CT} + \vec{F}_{TC} = 0 \]
• Tension, cords, pulleys

Ideal cord: massless, unstretchable

Ideal pulley: massless, frictionless

The forces at the two ends are always equal in magnitude.
Example:

- The object of mass $m_2$ hangs at the end of an ideal cord tied to the object of mass $m_1$.
- The object of mass $m_1$ is placed on a frictionless inclined plane.
- The cord is wrapped around an ideal pulley attached to the plane.
- Find the acceleration $\vec{a}_2$. 
Step 1: free body diagram for object 1

\[
\vec{F}_{\text{net}1} = m_1 \vec{a}_1
\]
\[
\vec{F}_{\text{net}1} = \vec{T}_1 + \vec{F}_{g1} + \vec{F}_N
\]
\[
(F_N)_x = 0, \quad (F_{g1})_x = -m_1 g \sin \alpha
\]
\[
T_1 - m_1 g \sin \alpha = m_1 a_{1x}
\]
\[
a_{1y} = 0 \quad \text{(no motion \perp to plane)}
\]
\[
m_1 a_{1x} = T_1 - m_1 g \sin \alpha
\]
Step 2: free body diagram for object 2

\[
\vec{F}_{\text{net}2} = m_2 \vec{a}_2
\]

\[
\vec{F}_{\text{net}2} = \vec{T}_2 + \vec{F}_{g2}
\]

\[
(F_{\text{net}2})_x = 0 \quad (F_{\text{net}2})_y = T_2 - m_2 g
\]

\[
m_2 a_{2x} = 0 \quad m_2 a_{2y} = T_2 - m_2 g
\]
Step 3: put equations together

\[ m_1 a_{1x} = T_1 - m_1 g \sin \alpha \]

\[ m_2 a_{2y} = T_2 - m_2 g \]

Can we solve?

Need extra conditions!

\[ T_1 = T_2 \]

\[ (\Delta x)_1 = - (\Delta y)_2 \text{ at all times} \]

\[ a_{1x} = - a_{2y} \]
\[ m_1 a_{1x} = T_1 - m_1 g \sin \alpha \quad \text{Can we solve?} \]

\[ m_2 a_{2y} = T_2 - m_2 g \quad \text{Yes!} \]

\[ T_1 = T_2 \]

\[ a_{1x} = -a_{2y} \]

\[ (m_1 + m_2) a_{2y} = m_1 g \sin \alpha - m_2 g \]

\[ a_{2y} = \frac{m_1 g \sin \alpha - m_2 g}{m_1 + m_2} \]
Which way will box 2 move? Up or down?

\[ a_{2y} = \frac{m_1 g \sin \alpha - m_2 g}{m_1 + m_2} \]

\[ a_{2y} > 0 \iff m_1 \sin \alpha > m_2 \quad \text{Up} \]

\[ a_{2y} < 0 \iff m_1 \sin \alpha < m_2 \quad \text{Down} \]

\[ a_{2y} = 0 \iff m_1 \sin \alpha = m_2 \quad \text{No motion (if initial velocity is 0)} \]