Problem 1.

A cart of mass \( m \) moves with speed \( v \) as it approaches a cart of mass \( 3m \) that is initially at rest. The spring is compressed during the head-on collision (see the figure below).

(a) What is the speed of the cart with mass \( 3m \) at the instant of maximum spring compression assuming conservation of energy?

(b) How would your answer differ if energy is not conserved?

(c) What is the final velocity of the heavier cart after a long time has passed, if energy is conserved?

(d) Give the final velocity of the heavier cart in a completely inelastic collision.
Problem 2.

A pendulum consisting of a mass $m$ connected to the pivot $O$ with a string of length $l$ is released from rest in a horizontal position. A nail a distance $d$ below the pivot causes the mass to move along the pass indicated by the dotted line. Find the minimum distance $d$ in terms of $l$ such that the mass will swing completely round in the circle shown in the figure below.

Note: The velocity of the mass $m$ in the top-most point of the circle is not zero. To find the critical condition for the mass to swing completely around the nail, think about the tension of the string: Can it be negative at any point on the trajectory?
Problem 3.

The two flywheels in the figure below are on parallel frictionless shafts but initially do not touch. The larger wheel rotates at \(f=2000\) rev/min while the smaller wheel is at rest. The two parallel shafts are moved until the wheels touch each other. Find the angular velocity of the second wheel after equilibrium is established (i.e. no further sliding at the point of contact), given that \(R_1 = 2R_2\), \(I_1 = 16I_2\), where \(I_1\) and \(I_2\) are the moments of inertia of the two wheels.

Note: The energy is not conserved here (there are friction forces, and the wheels initially slide with respect to each other). We solved a very similar problem in class once.
Problem 4.

A cone of height $h$ and base radius $R$ is constrained to rotate about its vertical axis, as shown in the figure below. A thin, straight groove is cut in the surface of the cone from apex to base as shown. The cone is set rotating with initial angular velocity $\omega_0$ around its axis, and a small (point-like) block of mass $m$ is released at the top of the frictionless groove and is permitted to slide down under gravity. Assume that the block stays in the groove, and that the moment of inertia of the cone about its axis is $I_0$.

(a) What is the angular velocity of the cone when the block reaches the bottom?
(b) Find the speed of the block (the magnitude and the direction) in the laboratory reference frame as it leaves the cone. You can give your answer in terms of the sum of the velocities parallel and perpendicular to the groove.
Problem 5.

Calculate the minimum coefficient of friction necessary to keep a thin circular ring from sliding as it rolls down a plane inclined at an angle $\theta$ with respect to the horizontal plane.

**Hint:** You may find it useful to use Newton equations, as well as the torque equation written with respect to the center-of-mass axis.
The captain of a small boat becalmed in the equatorial doldrums decides to resort to the expedience of raising an anchor to the top of the mast. The boat will begin to move.
   a) Why will the boat begin to move?
   b) In which direction will it move?
Problem 7

A comet in an orbit about the sun has a velocity 10 km/sec at aphelion and 80 km/sec at perihelion (see the figure below). If the earth's velocity in a circular orbit is 30 km/sec and the radius of its orbit is $1.5 \times 10^8$ km, find the aphelion distance $R_a$ for the comet. Your answer should contain only the numerical values given in the text of the problem.

*Hint:* one possible way to solve this problem is to use energy and angular momentum conservation laws for the comet.

![Diagram of comet orbit with velocities and distances labeled: Ua, Up, Ra, Rp]