The invention of the transistor & the discovery of semiconductor electronics was one of the transforming events of the 20th century, and it occurred not twenty miles from here, at Bell Labs in Murray Hill.

Today we’re going to learn about two important consequences of the energy band model of electrons in matter—

the free electron model of metals & the energy gap model of semiconductors.
**ATOM**
Discrete levels

**METAL**
High density
FAST ELECTRONS

**SEMICONDUCTOR**
Low density
TUNABLE CONCN.
of electrons (n)
& holes (p)

CONDUCTION

SMALL GAP

holes
42.5 Free Electron Model

A) Fermi Function

\[ f(E) = \text{probability state occupied} \]

\[ f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} \]

For example, at 1000K, what is the probability that a state 0.1eV above the Fermi energy of a metal is occupied?

\[ f = \frac{1}{e^{0.1 \times 1.6 \times 10^{-19} \frac{\text{J}}{1.38 \times 10^{-23} \times 1000}} + 1} = 0.24 = 24\% \]
B) DENSITY OF STATES

The number of states/unit energy = DENSITY OF STATES

\[ g(E) = \frac{(2m)^{3/2}}{2\pi^2 \hbar^3} \sqrt{V E} \]

\[ k_x = \frac{2\pi}{\lambda_x} = n_x \frac{\pi}{L} \]

\[ k^3 = \frac{\pi^2}{L^2} \left( n_x^2 + n_y^2 + n_z^2 \right) \]

\[ E = \frac{h^2 \pi^2}{2mL^2} \left( n_x^2 + n_y^2 + n_z^2 \right) \]

\[ E = \frac{h^2 \pi^2}{2mL^2} n_{rs}^2 \]
Inside: each state filled by two electrons
outside: states empty

volume of \frac{1}{8}th sphere
\[ = \frac{1}{8} \times \frac{4\pi}{3} n_{rs}^3 = \frac{\pi}{6} n_{rs}^3 \]

\# states = \# up + \# down
\[ = 2 \times \frac{\pi}{6} n_{rs}^3 = \frac{\pi}{3} n_{rs}^3 \]

\[ n = \frac{\pi}{3} n_{rs}^3 = \frac{\pi}{3} \times \left[ \left( \frac{2mL^2}{\hbar^2} E \right)^{1/2} \right]^3 = \frac{(2m)^{3/2} L^3 E^{3/2}}{3\pi^2 \hbar^3} \]

\[ = \left[ \frac{(2m)^{3/2}}{3\pi^2 \hbar^3} \right] V E^{3/2} \]

\[ \frac{dn}{dE} = g(E) = \left( \frac{(2m)^{3/2}}{2\pi^2 \hbar^3} \right) V E^{3/2} \]
C) ELECTRON DENSITY

\[ g(E) = \# VE^{1/2} \]

\[ dN = g(E)f(E) \, dE \]

\[ = \# VE^{1/2} \frac{1}{e^{(E-E_F)/k_B T} + 1} \]

\[ = 1 \quad E < E_F \]

\[ = 0 \quad E > E_F \]

\[ N = \int_0^{E_F} \# VE^{1/2} \, dE = 2 \# V E_F^{3/2} = \frac{2}{3} \left( \frac{2m}{2 \pi^2 k^3} \right)^{3/2} V E_{F_0}^{3/2} \]

\[ E_{F_0} = \left[ \frac{3 \pi^2 k^3}{(2m)^{3/2}} \right]^{2/3} \left( \frac{N}{V} \right)^{2/3} \]

\[ \frac{N}{V} = \left[ \frac{(2m)^{3/2}}{3 \pi^2 k^3} \right] E_{F_0}^{3/2} = \text{density of free electrons} \]

\[ \neq \text{density of electrons.} \]
e.g. Copper. Density of free electrons is \( N/V = 8.45 \times 10^{28} \, \text{m}^{-3} \).

What is the Fermi energy in electron volts?

\[
\left( \frac{3\pi^2 k^3}{(2m)^{3/2}} \right)^{2/3} = \left[ \frac{3 \times \pi^2 \times (1.05 \times 10^{-34})^3}{(2 \times 9.11 \times 10^{-31})^{3/2}} \right]^{2/3} = 5.84 \times 10^{-38} \, \text{J/m}^2
\]

\[
E_F \_0 = (5.84 \times 10^{-38}) \times (8.45 \times 10^{28})^{2/3} = 1.125 \times 10^{-18} \, \text{J}
\]

\[
= 7.03 \, \text{eV}
\]

How fast is an electron with this energy moving?

\[
\frac{1}{2} m v_F^2 = E_F
\]

\[
v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2 \times 1.125 \times 10^{-18}}{9.1 \times 10^{-31}}} \, \text{m/s} = 1.6 \times 10^6 \, \text{m/s}
\]

0.5% of the speed of light.
D) Average energy

\[ E_{av} = \frac{E_{\text{TOTAL}}}{N} = \frac{1}{N} \int_{0}^{E_{F0}} \# \ V E^{1/2} \times E \, dE \]

\[ = \frac{2}{5} \left( \frac{E_{F0}}{E_{F0}} \right)^{5/2} = \frac{2}{3} \left( \frac{E_{F0}}{E_{F0}} \right)^{3/2} = \frac{2}{5} E_{F0}. \]

This average energy is much, much greater than the thermal energy \( \frac{3}{2} k_{B} T \). At room temperature \( \frac{3}{2} k_{B} T = 0.04 \text{eV} \).
42.6 SEMICONDUCTORS

- Resistivity intermediate between metal & insulator.
- Small gap, typically less than 1 eV.

Hole concept

"Electronic equivalent of antimatter"!

hole = empty state in valence band

Im purifies

- donor: impurity with one more e− \( Z' = Z + 1 \)
- acceptor: impurity with one less e− \( Z' = Z - 1 \)

Ge \( 4s^2 4p \)
As \( 4s^2 4p^2 \) \( Z = 33 \)
\[ E_d = 0.01 \text{eV} \]

\[ E_d \ll \frac{1}{4^2}(13.6) = 0.85 \]

because of screening.

"n-type semiconductor"

Whereas for a p-type, e.g. Ga 4s^2 impurity. (\( t = 31 \))

"p-type semiconductor"
42.7 p-n Junctions

Basic workhorse of semiconductor electronics, LEDs, solar panels, semiconductor lasers.

\[ I = I_s \left( e^{\frac{eV}{k_B T}} - 1 \right) \]

\[ I_{\text{recomb}} = I_s e^{\frac{eV}{k_B T}} \]

\[ I_{\text{generation}} = -I_s \]
These events are the mini electronic analog of processes occurring inside clouders, and in cosmic ray events.