16. SPIN + EXCLUSION PRINCIPLE

In lecture 15 we focussed on the Zeeman effect.

Today we will see that electrons behave like tiny spinning tops. When a magnetic field is applied - the spin can point "up" or "down" and has a $z$-component of angular momentum equal to $\pm \hbar/2$.

We'll go onto discuss the exclusion principle - which is the basis of chemistry & the periodic table.

At the end of the lecture we shall discuss X-ray lines & isoelectronic.
41.3 Electron Spin

Stern-Gerlach (1922)

"up" \( S_z = \frac{1}{2} \)

"down" \( S_z = -\frac{1}{2} \)

Anomalous Zeeman effect.

These two experiments are a result of electron spin. Electrons behave as elementary "tops".

\[ S = \hbar \sqrt{\frac{1}{2}(1+\frac{1}{2})} = \frac{3}{4} \hbar \]

Magnitude of spin angular momentum

\[ S_z = m_s \hbar = \pm \frac{1}{2} \hbar \]

Components
\[ M_z = -2.00232 \frac{e}{2m} S_z \]

relativistic version of Schrödinger eqn

\[ \equiv \text{DIRAC EQN} \]

"Quantum Electrodynamics"

GQED

\[ 2.0023193043737(82) \]

\[ M_z = -2 \left( \frac{e}{2m} \right) m_s \hbar = -2M_B m_s \quad = \pm M_B \]

Energy in a magnetic field

\[ U = -M_z B = 2(M_B B) m_s = \pm (M_B B) \]

\[ \uparrow +M_B \quad S_z = \pm \frac{1}{2} \]

\[ \downarrow -M_B \quad S_z = -\frac{1}{2} \]
An electron absorbs a photon to flip from spin down to spin up in a magnetic field of 10 Tesla. What is the energy and frequency of the photon?

\[ \Delta E = 2 \mu_B B \]

\[ = 2 \times 5.788 \times 10^{-5} \text{eV/T} \times 10 \]

\[ = 1.15 \times 10^{-3} \text{eV} \]

\[ = 1.15 \text{ meV} \]

\[ h f = \Delta E \quad \Rightarrow \quad f = \frac{1.15 \times 10^{-3} \text{eV}}{4.14 \times 10^{-15} \text{eVs}} \]

\[ = 2.80 \times 10^{10} \text{ Hz} \]

\[ = 0.28 \text{ GHz} \]

This is the basis of spin resonance.
The Na D-line doublet is a result of spin-orbit coupling between the $l=1$ orbital angular momentum and the $S=1/2$ spin angular momentum.
4.1.4 **Many Electrons + Pauli Exclusion Principle**

Complex atom

\[ U(r) = -\frac{Ze^2}{4\pi \varepsilon_0 r} \]

\[ e^2 \rightarrow Ze^2 \]

\[ E_n = -\frac{1}{(4\pi \varepsilon_0)^2} \frac{m Ze^4}{2\hbar^2} \left( \frac{1}{n^2} \right) \]

\[ = -\frac{Z^2}{n^3} (13.6) \text{ eV} \]

Each \( e^- \) – Four Quantum numbers

\[ n, l, m_e, m_s \]

\[ n \geq 1, \quad 0 \leq l \leq n-1, \quad |m_e| \leq l, \quad m_s = \pm \frac{1}{2} \]

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**NO TWO ELECTRONS CAN OCCUPY THE SAME QUANTUM STATE**

If all \( e^- \) in lowest state \( \rightarrow \) ALL ELEMENTS \( \sim \) SAME

No Chemistry.

\[ 1s \quad 2s \quad 2p \]

\[ 1s^2 \quad 2s^2 \quad 2p^3 \]
### Electron Shells

<table>
<thead>
<tr>
<th>n</th>
<th>l</th>
<th>m_l</th>
<th>Notation</th>
<th># states</th>
<th>Shell</th>
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<tr>
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<td>2</td>
<td>K</td>
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<td>2</td>
<td>L</td>
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<td>2p</td>
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<td>10</td>
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<td>4p</td>
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<td>N</td>
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<tr>
<td>2</td>
<td>-2,-1,0,1,2</td>
<td></td>
<td>4d</td>
<td>10</td>
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<tr>
<td>3</td>
<td>-3 --- +3</td>
<td></td>
<td>4f</td>
<td>14</td>
<td></td>
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</table>
Ne. $Z = 10$

Filled L shell - Chemically inert

$1s^2 2s^2 2p^6$

$2s \uparrow \downarrow$

$2p \uparrow \uparrow$

F $Z = 9$

Strong affinity to attract electrons + fill shell. Highly reactive

$F + e^- \rightarrow F^-$ FILLED SHELL
Na \( Z = 11 \)

\( 1s^2 \ 2s^2 \ 2p^6 \ 3s^2 \)

\( 3s \quad \uparrow \)

\( 2s \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow - 2p \)

\( 1s \quad \uparrow \quad \uparrow \)

\( \text{Filled shell + 1e}^- \)

\( Z_{\text{eff}} = 11 - 10 = 1 \)

\( E_n = -\frac{Z_{\text{eff}}^2 (13.6)}{n^2} \)

\( E_{3s} = -5.138 \text{eV} \quad 1.84 \)
\( E_{3p} = -3.035 \text{eV} \)
\( E_{3d} = -1.521 \text{eV} \quad \sim 1.0 \)
\( E_{4s} = -1.947 \text{eV} \quad 1.51 \)

3s penetrates into filled shell

\( \Rightarrow \text{higher } Z_{\text{eff}} \)

\( V_{\text{eff}} = -\frac{e^2 Z_{\text{eff}}}{4\pi \epsilon_0 r} \)

\( 11 - 2 - 8 = Z_{\text{eff}} = 1 \)
This is the basis of the Periodic Table.

\[ \begin{array}{cccc}
3s & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ \\
2s & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ \\
1s & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{H} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{He} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{Li} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{Be} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
1s & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
1s^2 & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
1s^22s & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
1s^22s^2 & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{FILLED SHELL.} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{CHEMICALLY INERT} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{loosely bound} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
2s \text{ e}^- & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\Delta E = 5.4 eV & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{alkali metal} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
2s & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
2s^2 & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
1s & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
1s^2 & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{B} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
1s^22s^22p & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
\text{C} & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\
1s^22s^22p^2 & \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_
4.15 X-Ray Spectra

\[ \lambda_{\text{min}} = \frac{hc}{eV_{\text{AC}}} = \frac{1240 \text{ nm - eV}}{E(\text{eV})} \]

\[ \text{e.g.} \quad 50kV \]
\[ \lambda_{\text{min}} = \frac{1240}{50 \times 10^3} = 2.48 \times 10^{-2} \text{ nm} \]
\[ = 24.8 \text{ pm} \]

\[ Q = (z-1)e \]

\[ E_i = -\frac{(z-1)^2(13.6)}{2^2} = -(z-1)^2(3.42 \text{ eV}) \]

\[ E_f = -\frac{(z-1)^3(13.6)}{1^2} = -(z-1)^3(13.6 \text{ eV}) \]

\[ E_{K\alpha} = (z-1)^2 \times 10.2 \text{ eV} \]

\[ f = \frac{E}{h} = \frac{(z-1)^2 \times 10.2}{4.136 \times 10^{-15} \text{ eV-s}} = \frac{2.47 	imes 10^5 \text{ Hz} }{(z-1)^2} \]

Moseley 1913.
The conventional emission of light by an atom is called "spontaneous emission", and in this case, the direction & phase of the emitted light are random.

*STIMULATED EMISSION* is the emission of a photon in response to the arrival of a photon of matching frequency. In this process, the emitted photon has precisely the same frequency, phase & direction as the incoming photon. This "light amplification" effect is the basis of the operation of a laser.
Population Inversion

Under conventional thermal conditions, there is negligible excitation of atoms into their excited states. According to Boltzmann's distribution

\[ n_g = \text{# atoms in ground state} = A e^{-\frac{E_g}{k_B T}} \]
\[ n_{ex} = \text{# atoms in excited state} = A e^{-\frac{E_{ex}}{k_B T}} \]

\[ \frac{n_{ex}}{n_g} = e^{-\frac{(E_{ex} - E_g)}{k_B T}} \]

Example: if \( E_{ex} - E_g = 2 \text{eV} = 3.2 \times 10^{-19} \text{J} \approx 620\text{nm photon (visible)} \)

at \( T = 3000 \text{K} \)

\[ \frac{E_{ex} - E_g}{k_B T} = \frac{3.2 \times 10^{-19} \text{J}}{1.38 \times 10^{-23} \times (3000 \text{K})} = 7.73 \]

\[ e^{-\frac{(E_{ex} - E_g)}{k_B T}} = e^{-7.73} = 4.4 \times 10^{-4} . \]
To obtain a significant "population inversion", one must pump the atom into an excited state, e.g., He-Ne laser.

Diagram:

- Metastable state
- Impact collision
- Stimulated emission
- 632.8 nm laser
- 3p (18.70eV)
- Diffusion
- 16.7eV
- 2p

Additional text:

- Semiconductor laser (p-n)
- Chemical laser
- CO₂
- Maser - microwaves