2. **Images + Ray Optics**

Everything we see—everything, depends on ray optics.

From our reflection in the mirror to images of distant galaxies caught by the Hubble Space telescope—all depend on the way light from a distant object is manipulated so that it appears to come from an image. Understanding the relationship between the positions, sizes & orientations of the object & image is the topic of today's lecture, and we shall make extensive use of ray optics.
34.1 Object & image points

$P$— the position of a self-luminous, point object

$P'$— apparent origin of light rays, the image point.

If rays do not actually pass through the image point $P'$, it is said to be a virtual image.
\[ \triangle \text{PVB} \& \triangle \text{P'VB} \text{ are congruent} \]
\[ \Rightarrow s = s' \text{ for all points P \& P'}. \]

### Sign Rules

|                | Same side as incoming light | Same side as outgoing light | \( s > 0 \)  
|----------------|-----------------------------|----------------------------|---------------------------
| Object         |                             |                            | (otherwise \( s < 0 \)) |
| Image          |                             |                            | \( s' > 0 \)          
| (otherwise \( s' < 0 \)) |
| Radius of curvature | Center of curvature is on the same side as outgoing light |
|                |                             |                            | \( R > 0 \)          
|                |                             |                            | (otherwise \( R < 0 \)) |
e.g. Plane mirror - virtual image on opposite side to outgoing light \( s' < 0 \)

\[ s' = -s \]

**Magnification**

\[ m = \frac{y'}{y} \]

\( m' > 0 \) erect image.

Mirror reverses back to front
34.2 REFLECTION @ SPHERICAL SURFACE

\[ \frac{1}{S} + \frac{1}{S'} = \frac{2}{R} \]

object-image reln, spherical mirror

\[ m = -\frac{S'}{s} \]

lateral magnification, spherical mirror.

\[ \alpha + \theta = \phi \]
\[ \phi + \theta = \beta \]

\[ \Rightarrow \alpha + \beta = 2\phi \] (1)

\[ \frac{h}{s - \delta} = \tan \alpha \quad \frac{h}{R - \delta} = \tan \phi \quad \frac{h}{s' - \delta} = \tan \beta \]

\[ \alpha \approx \tan \alpha \quad \phi \approx \tan \phi \quad \beta = \tan \beta \quad s \gg \delta \quad s' \gg \delta \quad R \gg \delta \]

\[ \alpha + \beta = \frac{h}{s} + \frac{h}{s'} = \frac{2h}{R} \Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \]
The object-image relation is an approximate formula, valid for small angles. All rays from $P \rightarrow P'$.

\[
\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \quad s' = \frac{R}{2}
\]

\[
f = \frac{R}{2} = \text{focal length.}
\]

\[
\frac{1}{s} + \frac{1}{\infty} = \frac{2}{R} \quad s' = \frac{R}{2}
\]

Image at infinity — object at focal point.

Object at infinity — image at focal point.

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \boxed{f = \frac{R}{2}}
\]

Larger angles — $P'$ moves inward towards mirror ⇒ spherical aberration.
n.b.  
- Solar furnaces: place furnace or generator at F.P. of large mirror

- Initial problems with Hubble Space Telescope in 1993 were caused by spherical aberration.
Image of a concave spherical mirror

\[ m = \frac{y'}{y} \]

\[ \text{PVQ & P'VA' are similar } \Rightarrow \frac{y}{s} = -\frac{y'}{s'} \quad (y' < 0) \]

\[ m = \frac{y'}{y} = -\frac{s'}{s} \]

lateral magnification, spherical mirror.
e.g. 34.1

Concave mirror forms image of filament which is 10 cm from mirror at a distance of 3 m.

a) What is radius of curvature?

b) What is the height of the image if the filament is 5 mm high?

\[
\begin{align*}
a) \quad s &= 10 \text{ cm}, \quad s' = 300 \text{ cm} \\
\frac{1}{s} + \frac{1}{s'} &= \frac{1}{10} + \frac{1}{300} = 0.1033 = \frac{2}{R} \\
R &= \frac{2}{0.1033} = 19.4 \text{ cm} \\
b) \quad \frac{y}{s} &= -\frac{y'}{s'} \\
y' = \frac{y}{s} s' = -\left(\frac{s'}{s}\right)y &= -\left(\frac{300}{10}\right) \times 5 \text{ mm} \\
&= -30 \times 5 \\
&= -150 \text{ mm}
\end{align*}
\]
Convex Mirrors

1 \over s + 1 \over s' = 2 \over R

m = \gamma' / \gamma = -s' / s

But now R < 0

1 \over \infty + 1 \over s' = 2 \over R

s' = -|R| \over 2

\leftarrow R \rightarrow

\text{negative}

\leftarrow R < 0

1 \over s + 1 \over \infty = 2 \over R

s = R = -|R| \over 2
Santa Claus reflected in an ornament

$d_{\text{ornament}} = 7.2\text{cm}$

$h_{\text{Santa}} = 1.6\text{m}, \text{ standing } s = 0.75\text{m from ornament}$

How high is the image of Santa?

$$|R| = \frac{7.2}{2} = 3.6\text{cm}$$

$$f = \frac{R}{2} = -1.8\text{cm}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{-1}{1.8}$$

$$\frac{1}{75} + \frac{1}{s'} = \frac{-1}{1.8} \Rightarrow \frac{1}{s'} = \frac{-1}{1.8} - \frac{1}{75}$$

$$s' = -1.758\text{cm}$$

$$m = -\frac{s'}{s} = \frac{1.758}{75} = +0.0234$$

$$y' = y m = 0.0234 \times 1.6\text{m} = 0.0374\text{m}$$

$$y' = 3.74\text{cm}$$

virtual & upright.
Graphical methods (mirrors) "Papo Rave"

1. Parallel ray to axis → passes through focal point F on reflection.

2. Focal ray, passing through focal point → is parallel on reflection.

3. Radial ray through center of curvature C, bounces back along radial path through C.

4. Vertex ray, passing through vertex, is reflected forming equal angles with the optical axis.
34.3 Refraction at a Spherical Surface.

\[ \frac{n_a}{S} + \frac{n_b}{S'} = \frac{n_b - n_a}{R} \]

Object-image relation:
Spherical refracting surface.

- \( \Theta_a = \alpha + \varphi \)
- \( \varphi = \Theta_b + \beta \)
- exterior \( \angle \) = sum of opposite interior \( \angle \)s.

- \( n_a \sin \Theta_a = n_b \sin \Theta_b \Rightarrow n_a \Theta_a = n_b \Theta_b \Rightarrow n_a (\alpha + \varphi) = n_b (\varphi - \beta) \)

- \( \tan \alpha = \frac{h}{S + \delta} \quad \tan \varphi = \frac{h}{R - \delta} \quad \tan \beta = \frac{h}{S' - \delta} \)

\( \Rightarrow \alpha = \frac{h}{S} \quad \varphi = \frac{h}{R} \quad \beta = \frac{h}{S'} \Rightarrow n_a \left( \frac{h + \frac{h}{S}}{R} \right) = n_b \left( \frac{h - \frac{h}{S'}}{R} \right) \)
Snell's law becomes

\[ n_a \sin \theta_a = n_b \sin \theta_b \]

\[ n_a \left( \frac{y}{s} \right) = n_b \left( \frac{-y'}{s'} \right) \]

\[ \Rightarrow m = \frac{y'}{y} = -\left( \frac{s'/n_b}{s/n_a} \right) \]

\[ n_a \frac{s}{s'} + n_b \frac{-s}{s'} = 0 \]

\[ s' = -s \left( \frac{n_b}{n_a} \right) \]

Swimming pool \[ s' = -s \left( \frac{n_{\text{air}}}{n_{\text{wat}}} \right) = -\frac{s}{1.33} \]

\[ R = \infty \]

**e.g.** Plane Surface