The invention of the transistor & the discovery of semiconductor electronics was one of the transforming events of the 20th century, and it occurred not twenty miles from here, at Bell Labs in Murray Hill.

Today we're going to learn about two important consequences of the energy band model of electrons in matter — the free electron model of metals & the energy gap model of semiconductors.
ATOM
discrete levels

SOLID
energy bands: electrons delocalize

No carriers

\[ \text{CONDUCTION } E_c \]
\[ \text{VALENCE } E_v \]

INSULATOR

\[ E_g \]

Low density.

\[ \text{CONDUCTION BAND} \]
\[ \text{VALENCE BAND} \]

\[ \text{SEMICONDUCTOR (small energy gap)} \]
\[ \text{Tunable conc. of carriers} \]

\[ P = \frac{N_c}{N_v} \sim e^{-\frac{(E_c-E_v)}{k_B T}} \]

\[ \text{DIAMOND } E_g = 5.5 \text{ eV} \]
\[ T = 300 \text{ K} \]

\[ P \sim e^{-\frac{213}{k_B T}} \sim 10^{-93} \]

Good insulator

High Density

\[ \text{CONDUCTION} \]
\[ \text{FERMI E.} \]

METAL
Number of Conduction Electrons

\[
\text{(no. (cond. e\textsuperscript{-})) = (\# (atoms) (\# valence electrons/atom))}
\]

\[n = \frac{\# \text{ cond e}\textsuperscript{-}}{\text{volume} V} = \left(\frac{g_{\text{Na}}}{M}\right) \times n_{\text{vol}}\]

\[
\# \text{atoms} = \frac{\text{Sample mass } M_{\text{sam}}}{\text{atomic mass}} = \frac{M_{\text{sam}}}{(\text{molar mass } / \text{NA})}
\]

\[= \left(\frac{\text{density } \times \text{volume}}{\text{molar mass } M / \text{NA}}\right) = \frac{g V_{\text{Na}}}{M}\]

\[
\text{e.g. Mg, density} = 1.738 \text{g/cm}^3 = 1.738 \times 10^3 \text{kg/m}^3
\]

\[
\text{M} = 24.312 \text{g/mol} = 24.3 \times 10^{-3} \text{kg/mol}
\]

\[
\text{How many electrons in } 2 \text{cm}^3 = 2 \times 10^{-6} \text{ m}^3?
\]

\[
\# \text{atoms} = \frac{1.738 \times 10^3 \times 2 \times 10^{-6} \text{ m}^2}{(24.3 \times 10^{-3} / 6.02 \times 10^{23})} = \frac{2.09 \times 10^{21} \text{ atoms kg/mol}}{24.3 \times 10^{-3} \text{ kg/mol}} = 8.6 \times 10^{22} \text{ atoms}
\]

\[
\# \text{electrons} = (8.6 \times 10^{22} \text{ atoms}) \times \left(2 \text{ e}^{-}/\text{atom}\right) = 1.72 \times 10^{23} \text{ electrons.}
\]

\[
n = \frac{1.72 \times 10^{23}}{2 \times 10^{-6}} = 8.5 \times 10^{29} \text{ e/m}^3
\]
\[ p(E) = \begin{align*}
\frac{1}{e^{(E-E_F)/k_BT} + 1}
\end{align*} \]

**Example:** At 1000K, what is the probability that a state 0.1eV above the Fermi energy of a metal is occupied?

\[ p = \frac{1}{e^{0.1 \times 1.6 \times 10^{-19} J / 1.38 \times 10^{-23} \times 1000} + 1} = 0.24 = 24\% \]
B) **Density of States**

The number of states/unit energy = Density of States

\[ g(E) = N(E) \, V \]

\[ g(E) = \frac{(2m)^{3/2}}{2\pi^2 \hbar^3} \, V \, E^{1/2} \]

\[ k_x = \frac{2\pi}{\lambda_x} = n_x \, \frac{\pi}{L} \]

\[ k^2 = \frac{\pi^2}{L^2} \left( n_x^2 + n_y^2 + n_z^2 \right) \]

\[ E = \frac{\hbar^2 \pi^2}{2mL^2} \left( n_x^2 + n_y^2 + n_z^2 \right) \]

\[ E = \frac{\hbar^2 \pi^2}{2mL^2} \, n_{rs} \]
Inside: each state filled by two electrons
outside: states empty

\[ \text{volume of } \frac{1}{8} \text{th sphere} = \frac{1}{8} \times \frac{4\pi}{3} r^3 = \frac{\pi}{6} n_{rs}^3 \]

\[ E = \frac{\hbar^2}{2m} \left( \frac{\pi}{L} n_{rs} \right)^2 \]

\[ \text{# states} = \text{# up} + \text{# down} = 2 \times \frac{\pi}{6} n_{rs}^3 = \frac{\pi}{3} n_{rs}^3 \]

\[ n = \frac{\pi}{3} n_{rs}^3 = \frac{\pi}{3} \times \left[ \left( \frac{2m L^2}{\hbar^2 \pi^2 E} \right)^{\frac{1}{2}} \right]^3 = \left( \frac{2m}{3 \pi^2 \hbar^3} \right) \frac{L^3 E}{E^{3/2}} \]

\[ \frac{dn}{dE} = g(E) = \left( \frac{(2m)^{3/2}}{2\pi^3 \hbar^3} \right) V E^{3/2} \]

\[ N(E) = g(E) = \frac{(2m)^{3/2}}{2\pi^3 \hbar^3} E^{1/2} \]
C) **Electron Density**

\[ N_0(E) = N(E)p(E) \]

\[ g(E) = \# \text{VE}^{1/2} \]

\[ dN = g(E)p(E)dE \]

\[
\begin{align*}
N &= \int_0^{E_F} \# \text{VE}^{1/2} \, dE \\
&= \frac{2}{3} \# \text{VE}_F^{3/2} = \frac{(2m)^{3/2}}{3\pi^2k^3} \text{VE}_F^{3/2}
\end{align*}
\]

\[
\begin{align*}
E_F &= \left[ \frac{3\pi^2k^3}{(2m)^{3/2}} \right]^{2/3} \left( \frac{N}{V} \right)^{2/3} \\
\frac{N}{V} &= \left[ \frac{(2m)^{3/2}}{3\pi^2k^3} \right] E_F^{3/2}
\end{align*}
\]

\[ = \text{density of free electrons} \]

\[ \neq \text{density of electrons} \]
e.g. Copper. Density of free electrons is \( \frac{N}{V} = 8.45 \times 10^{29} \text{ m}^{-3} \).

What is the Fermi energy in electron volts?

\[
\left[ \frac{3\pi^2 k^3}{(2\pi)^3} \right]^{2/3} = \left[ \frac{3 \times \pi^2 \times (1.05 \times 10^{-34})^3}{(2 \times 9.1 \times 10^{-31})^{3/2}} \right]^{2/3} = 5.84 \times 10^{-38} \text{ J m}^2
\]

\[
E_F = (5.84 \times 10^{-38}) \times (8.45 \times 10^{29})^{2/3} = 1.125 \times 10^{-18} \text{ J} = 7.03 \text{ eV}
\]

How fast is an electron with this energy moving?

\[
\frac{1}{2} m v_F^2 = E_F
\]

\[
v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2 \times 1.125 \times 10^{-18}}{9.1 \times 10^{-31} \text{ kg}}} = 1.6 \times 10^6 \text{ m/s}
\]

0.5% of the speed of light.
D) Average energy

\[ E_{av} = \frac{E_{\text{total}}}{N} = \frac{\int_{E_F}^{E_F} \#V E^{\frac{1}{2}} \times E \, dE}{\int_{0}^{E_F} \#V E^{\frac{1}{2}} \, dE} = \frac{\frac{2}{5} E_F^{5/2}}{\frac{2}{3} E_F^{3/2}} = \frac{3}{5} E_F \]

This average energy is much, much greater than the thermal energy \( \frac{3}{2} k_B T \). At room temperature

\[ \frac{3}{2} k_B T = 0.04 \text{eV} \]
42.6 SEMICONDUCTORS

- Resistivity intermediate between metal & insulator.
- Small gap, typically less than 1 eV.

Hole concept

“Electronic equivalent of antimatter”!

hole = empty state in valence band

Impurities

donor = impurity with one more e⁻ \( Z' = Z + 1 \)

acceptor = impurity with one less e⁻ \( Z' = Z - 1 \)

\[
\begin{align*}
\text{Ge} & \quad 4s^2 \quad 4p^2 \\
\text{Ge} & \quad 4s^2 \quad 4p^3 \\
\text{As} & \quad 4s^2 \quad 4p^3 \\
\text{Z} & = 33
\end{align*}
\]
Ed = 0.01eV

Conduction

\[ Ed \ll \frac{1}{4e^2} (13.6) = 0.85 \]

because of screening.

Electrical Field

Valence

"n-type semiconductor"

Whereas for a p-type e.g. Ga\(^{4s^24p^1}\), impurity. (Z = 31)

Electrical Field

Conduction

"stolen electron"

Acceptor hole

"p-type semiconductor"
42.7 p-n JUNCTIONS

Basic work-horse of semiconductor electronics, LEDs, solar panels, semiconductor lasers.

\[ I = I_s \left( e^{\frac{eV}{k_B T}} - 1 \right) \]

Diffusion Current

\[ I_{\text{diff}} = I_s e^{\frac{eV}{k_B T}} \]

Drift Current

\[ I_{\text{drift}} = -I_s \]
These events are the mini electronic analog of processes occurring inside colliders, and in cosmic ray events.