Photons and Matter Waves

38.1 At the dawn of the 20th century, classical physics was in trouble. The most elementary property of matter— that as it gets hotter it changes color from red, to white hot— the observation of a myriad of sharp emission lines in the spectra of elements— and the discovery of X-rays— none of these phenomena could be explained in terms of the classical theory of radiation.

The great discovery of this time was that
Radia
tion—electromagnetic radiation, including light—is not smooth and continuous, as assumed in Maxwell's theory—but rather, it is "grainy" and made up of tiny packets of energy called "quanta of energy".

It was Max Planck who first observed that the changing color of hot bodies can be understood if the energy of the radiation splits up into quanta of energy $E$, where

$$E = hf$$

$$h = 6.626 \times 10^{-34} \text{ Js}$$

It was Einstein who made the radical leap and
concluded that light must be made up of parhaes - which we call photons, each carrying energy \( h\nu \). Einstein made this proposal in 1905, but it took about twenty years before it was widely accepted.
Mysteries resolved by Quantum Hypothesis

- **Line Spectra**

  Why does each gas produce a discrete set of lines?

- **Photo-electric effect**

  (photon in, e\(^-\) out)

- **X-rays.**

  (e\(^-\) in, photon out)

\[ \lambda' > \lambda \quad \text{increase on scattering} \]
Blackbody Radiation

Hot → Hotter → White Hot

Hot matter emits a continuous spectrum of "blackbody" radiation.
38.2 Photoelectric Effect

\[ eV_0 = kE_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 \quad \text{does not depend on intensity} \]

\[ kE_{\text{max}} = \text{photon energy} - \text{work fn} = hf - \phi \]

Einstein, 1905.

The independence of the electron energy on intensity & the dependence of \( kE_{\text{max}} \) on frequency are accounted for by the photon concept.
• Photons have no rest mass but they are never at rest \( m = m_0 \gamma \) is finite even though \( m_0 = 0 \).

\[
\begin{align*}
E &= mc^2 \quad \{ \text{E = pc} \} \\
p &= mc
\end{align*}
\]

• \( P = \frac{E}{c} = \frac{\hbar f}{c} = \frac{\hbar}{\lambda} \)

• Physicists often use the electron volt:

\[
1 \text{eV} = 1.602 \times 10^{-19} \text{J}
\]

\[
\hbar = 6.626 \times 10^{-34} \text{Js} = 4.136 \times 10^{-15} \text{eVs}
\]
Sodium has a work function $\phi = 2.7 \text{eV}$.

a) What is the minimum energy photon required to produce a photocurrent?

b) What is the maximum electron energy produced by light of wavelength 400nm?

\[
\begin{align*}
\text{a)} & \quad hf_{\text{min}} = \phi = 2.7 \text{eV} \\
\text{b)} & \quad \frac{1}{2} mv^2_{\text{max}} = hf - \phi
\end{align*}
\]

\[
\begin{align*}
\lambda &= \frac{c}{f} = \frac{3 \times 10^8 \text{m/s}}{4 \times 10^{-7} \text{m}} = 0.75 \times 10^{15} \text{Hz} \\
\lambda &= 4.14 \times 10^{-15} \text{eVs} \times 0.75 \times 10^{15} = 3.11 \text{eV} \\
\frac{1}{2} mv^2_{\text{max}} &= 3.11 - 2.7 = 0.41 \text{eV}
\end{align*}
\]
X-rays are produced by the absorption of high energy electrons. Called "breaking radiation" or "brehamstrahlung".

\[ eV = \text{energy of incoming } e^- = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} \]

When X-rays scatter off an electron, they lose momentum and energy, which increases their wavelength. This process is called the "Compton Effect".

\[ (\lambda' - \lambda) = \frac{h}{mc} (1 - \cos \phi) \]

\[ \frac{h}{mc} = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} = 2.43 \times 10^{-12} \]
38.7 Compton Scattering + X-rays.

\[ eV = k\gamma = hf_{\text{max}} \]
\[ = \frac{hc}{\lambda_{\text{min}}} \]

\[ \lambda_{\text{min}} = \frac{hc}{eV} = \frac{6.63 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{m/s}}{1.6 \times 10^{-19} \text{V}} \]
\[ = 1.24 \times 10^{-6} \text{ m/V} \]

\[ V = 50,000 \text{V} \]
\[ \lambda = \frac{1.24 \times 10^{-6}}{50,000} = 2.5 \times 10^{-11} \text{m} = 0.25 \text{Å} \]

\[ \lambda' - \lambda = \frac{h}{m_c} (1 - \cos \theta) \]

\[ \frac{h}{m_c} = 2.43 \times 10^{-12} \text{m} \]
\[ \phi = 0 \]

\[ \Delta \lambda = \frac{h}{mc} \]
Proof of Compton Formula.

First note that the energy and momentum of the outgoing e\textsuperscript{-}
are

\[
\begin{align*}
\frac{E}{c} &= mc \gamma \quad \left\{ \begin{array}{l}
(\frac{E}{c})^2 - p_e^2 = m^2 c^2 \\
p_e = m u \gamma
\end{array} \right. \\
E &= mc^2 + hf - hf' \\
\Rightarrow \quad \frac{E}{c} &= mc + p - p' \\
\Rightarrow \quad \left(\frac{E}{c}\right)^2 &= (mc + p - p')^2 \\
\vec{p}_e &= \vec{p} - \vec{p}' \\
\Rightarrow \quad p_e^2 &= p^2 + p'^2 - 2pp' \cos \Theta
\end{align*}
\]

\[
\left(\frac{E}{c}\right)^2 - p_e^2 = (mc + p - p')^2 - (p^2 + p'^2 - 2pp' \cos \Theta)
= m^2 c^2 + 2mc(p - p') + 2pp' \cos (\Theta - 1)
= m^2 c^2 \quad \text{(by 1)}
\]

\[
\Rightarrow \quad mc(p - p') = pp' (1 - \cos \Theta)
\]

\[
\Rightarrow \quad \left(\frac{1}{p^2} - \frac{1}{p'}\right) = \frac{1}{mc} (1 - \cos \Theta) \quad \Rightarrow \quad \left(\frac{h}{p'} - \frac{h}{p}\right) = \frac{h}{mc} (1 - \cos \Theta)
\]

\[
\lambda' - \lambda = \frac{h}{mc} (1 - \cos \Theta)
\]
38.4 The Birth of Quantum Physics

Hot → Hotter → White Hot

Hot matter emits a continuous spectrum of "blackbody" radiation.

\[ S(\lambda) \, d\lambda = \text{intensity of radiation with wavelength between } \lambda \text{ and } \lambda + d\lambda \]

**Total Intensity**

\[ I = \int_0^\infty S(\lambda) \, d\lambda = \sigma \, T^4 \]

\[ \sigma = 5.67 \times 10^{-8} \, \text{W m}^{-2} \text{K}^{-4} \]

\[ \lambda_m \, T = 2.9 \times 10^{-3} \, \text{m-K} \]

Wien Displacement Law

Higher T → Smaller \( \lambda_m \)
Fitting waves into a box.

Rayleigh:

\[ S(\lambda) = \text{number of modes} \times k_B T \]

with wavelength \( \lambda \)

\[ = \frac{2\pi c}{\lambda^4} \ k_B T \]

- Can't explain \( \lambda_m \propto 1/T \)
- Leads to an ultraviolet catastrophe

\[ I = \int_0^\infty \frac{d\lambda}{\lambda^4} = \infty \]

Planck: energy in each mode is

He then deduced

\[ S(\lambda) = \frac{2\pi c}{\lambda^4} \times \frac{(hc/\lambda)}{e^{(hc/\lambda k_B T)} - 1} \]

\[ \lambda_m = \frac{hc}{kT} \times 4.97 \]

\[ I = \int S(\lambda) d\lambda = \sigma T^4 \]

\[ \sigma = \frac{2\pi^2 k_B^4}{15e^4 h^3} \]

Planck radiation law

Planck energies

\[ hf \sim k_B T \]
38.8 Ex

Sun T_{\text{surface}} \sim 5800K

a) Peak intensity \( \lambda_m = \frac{2.9 \times 10^{-3} \text{mK}}{5800K} = 0.5 \times 10^{-6} = 500 \text{nm} \)

b) Intensity \( I = \sigma T^4 \times (5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}) \times (5.8 \times 10^3)^4 \)

\[ = 6.42 \times 10^{-7} \text{W/m}^2 = 64.2 \text{MW/m}^2 \]

E) How much energy radiated between 622 \& 630 nm?

\[ \frac{hc}{\lambda K_B T} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{622 \times 10^{-9} \times 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}} \times 5800} = 4.0 \]

\[ I(\lambda) = \left( \frac{2\pi \times 3 \times 10^8}{\lambda^4} \right) \times K_B T \times \left( \frac{hc}{\lambda K_B T} \right) \left( \frac{1}{e^{hc/\lambda K_B T} - 1} \right) \]

\[ = \frac{2\pi \times 3 \times 10^8}{(622 \times 10^{-9})^4} \times (1.38 \times 10^{-23} \times 5800) \times \frac{4}{(e^{4} - 1)} \]

\[ = 1.26 \times 10^{34} \frac{\text{m}^{-3}}{\text{m}^2} \times (8.0 \times 10^{-20} \text{J}) \times 0.746 \]

\[ = 7.53 \times 10^{13} \text{W/m}^3 \]

\[ I(\lambda) \Delta \lambda = 7.53 \times 10^{13} \times 8 \times 10^{-9} = 6.0 \times 10^5 \text{ W} \]
LIGHT AS A PROBABILITY WAVE.

Which slit did photon travel through?

Any attempt to discern which slit the photon travelled through destroys the interference pattern.

\[ p \propto |E_1 + E_2|^2 \]

\[ p(x) \propto E(x)^2 \]

\[ p(\theta) \propto I(\theta) \propto E^2 \]

probability of photon arrival is proportional to intensity

Works with just single photons!

Light is alternatively viewed as a probability wave.