3. IMAGES FORMED BY REFRACTION:
   THIN LENSES.

The lense is the most widely used optical device.

On a hot sunny day, you have surely experimented with
magnifying glasses & observed how a lens can be used
to bum holes in paper, fry ants & even start a fire.

When you do this, you are imaging the surface of the
sun, on the paper, (or ant) that you are burning.

What then is the relation between the image & object
for a lens?
\[ \triangle OA'V \quad \text{and} \quad \triangle OA'V' \quad \text{are similar} \]

\[ \frac{y}{p} = \frac{-y'}{i} \]

\[ m = \frac{y'}{y} = \frac{-i}{p} \quad \text{(1)} \]

Also \[ \triangle I'A'F_2 \quad \text{and} \quad \triangle AF_2V \quad \text{are similar} \]

\[ \Rightarrow \quad \frac{y'}{f} = \frac{-y'}{i-f} \quad \Rightarrow \quad \frac{y'}{y} = -\frac{i-f}{f} \quad (2) \]

Combining (1) \& (2) gives

\[ \frac{i}{p} = \frac{i-f}{f} \quad \Rightarrow \quad \frac{f}{p} = 1 - \frac{f}{i} \]

\[ \Rightarrow \quad \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad \text{thin lens} \quad \text{o-image reln.} \]
But how is the focal length related to the dimensions of the lens? Here we can use our basic refractive equation

\[ \frac{n_a}{p} + \frac{n_b}{i} = \frac{n_b - n_a}{r} \]

but apply it to a lens which has two radii of curvature

The light refracting off the second surface is focused at a distance \( i \) to the right of the surface, and behaves as an object at position \( p' = -i \) for the second surface.
\[
\frac{1}{p} + \frac{n}{i'} = \frac{n-1}{r_1} \tag{1}
\]

The image at \(i'\) behaves as a **virtual** object for the second surface, so in

\[
\frac{n}{p'} + \frac{1}{i} = \frac{1-n}{r_2} \quad \Rightarrow \quad -\frac{n}{i'} + \frac{1}{i} = \frac{1-n}{r_2} \tag{2}
\]

\((p' = -i')\)

(1) & (2)

\[
\frac{1}{p} + \frac{1}{i} = (n-1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

**Lensmaker equation**

\[
\frac{1}{f} = (n-1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right)
\]
\[ |r_1| = |r_2| = 10 \text{ cm} \]

\[ n = 1.52 \]

**What is the focal length?**

- \( C_1 \) to right \( \Rightarrow r_1 > 0 \)
  \[ r_1 = +10 \text{ cm} \]
- \( C_2 \) to left \( \Rightarrow r_2 < 0 \)
  \[ r_2 = -10 \text{ cm} \]

\[
\frac{1}{f} = (1.52 - 1) \left( \frac{1}{10 \text{ cm}} - \frac{1}{-10 \text{ cm}} \right) = 0.52 \times \frac{2}{10 \text{ cm}} = \frac{0.52}{5 \text{ cm}}
\]

\[ \Rightarrow f = 9.6 \text{ cm} \]
34.4c Graphical Methods for Lenses.

1. Parallel ray $\rightarrow F_2$
2. Ray through center
3. Ray through $F_1 \rightarrow$ parallel
Light rays converging on lens.

\[ \frac{i}{p} \text{ bigger} \]

\[ \frac{i}{p} = \infty \]

Image at infinity

Image virtual & larger than object
(magnifying glass)
\( f = 20 \text{ cm} \)

Calculate \( m \) & \( i \)

a) \( s = 50 \text{ cm} \)
\[
\frac{1}{50} + \frac{1}{i} = \frac{1}{20} \quad \Rightarrow \quad i = 33.3 \text{ cm} \quad m = \frac{33.3}{50} = 0.66
\]

b) \( s = 20 \text{ cm} \)
\[
\frac{1}{20} + \frac{1}{i} = \frac{1}{20} \quad \Rightarrow \quad i = 200 \text{ cm} \quad m = 1
\]

c) \( s = 15 \text{ cm} \)
\[
\frac{1}{15} + \frac{1}{i} = \frac{1}{20} \quad \Rightarrow \quad i = -60 \text{ cm} \quad m = \frac{60}{15} = 4
\]

d) \( s = -40 \text{ cm} \)
\[
\frac{1}{-40} + \frac{1}{i} = \frac{1}{20} \quad \Rightarrow \quad i = 13.3 \text{ cm} \quad m = \frac{-13.3}{-40.0} = 0.33
\]

3) Image of an image.

![Diagram of optical system with focal lengths and distances marked]

\( f = 8.0 \text{ cm} \quad f > 6.0 \text{ cm} \)

Find position, size & orientation of image produced.

\[
\frac{1}{12} + \frac{1}{i_1} = \frac{1}{8.0} \quad \Rightarrow \quad i_1 = +24 \text{ cm} \quad \Rightarrow \quad p_2 = 12 \text{ cm}
\]

\[
\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{6.0} \quad \Rightarrow \quad i_2 = 12 \text{ cm} \quad m_{2} = \frac{-12}{12} = -1
\]

\[
m_{TOT} = m_1m_2 = +2
\]
\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]

Two lenses: image of second is object of first.

\[ \leftarrow f_1 \rightarrow \quad \leftarrow f_2 \rightarrow \leftarrow f_2 \rightarrow \]

\[ \leftarrow i \quad d \rightarrow \leftarrow i' \rightarrow \]

e.g. \( p = 20 \text{ cm} \) \( f_1 = 10 \text{ cm} \) \( f_2 = 5 \text{ cm} \)
\( d = 40 \text{ cm} \)

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]
\[ \frac{1}{20} + \frac{1}{i} = \frac{1}{10} \]
\[ \frac{1}{i} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \quad \Rightarrow \quad i = 20 \text{ cm} \]
\( m_1 = -1 \)

\[ p_2 = d - i = 40 - 20 = 20 \text{ cm} \]

\[ \frac{1}{i} + \frac{1}{i'} = \frac{1}{5} \]
\[ \frac{1}{i'} = \frac{1}{5} - \frac{1}{20} = \frac{4 - 1}{20} = \frac{3}{20} \]
\[ \Rightarrow \quad i' = \frac{20}{3} = 6.6 \text{ cm} \]

\( m_2 = -\frac{1}{3} \) \( m_1m_2 = +\frac{1}{3} \).