3. IMAGES FORMED BY REFRACTION: THIN LENSES.

The lens is the most widely used optical device. On a hot sunny day, you have surely experimented with magnifying glasses & observed how a lens can be used to bum holes in paper, fry ants & even start a fire. When you do this, you are imaging the surface of the sun, on the paper, (or ant) that you are burning. What then is the relation between the image & object for a lens?
\[ \Delta QAV \quad \& \quad \Delta Q'VI \quad \text{are similar} \]

\[ \frac{y}{p} = \frac{-y'}{i} \Rightarrow \frac{y'}{y} = \frac{-i}{p} \quad \text{(1)} \]

Also \( \Delta l'Q'F_2 \quad \& \quad \Delta AF_2V \quad \text{are similar} \)

\[ \Rightarrow \frac{y'}{f} = -\frac{y'}{i-f} \quad \Rightarrow \quad \frac{y'}{y} = -\frac{i-f}{f} \quad \text{(2)} \]

Combining (1) & (2) gives

\[ \frac{i}{p} = \frac{i-f}{f} \Rightarrow \frac{f}{p} = 1 - \frac{f}{i} \]

\[ \Rightarrow \quad \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad \text{thin lens} \quad \text{o-image reln.} \]
34.4 Lensmaker Equation

But how is the focal length related to the dimensions of the lens? Here we can use our basic refraction equation

\[ \frac{n_a + n_b}{p} + \frac{n_b - n_a}{i} = \frac{n_b - n_a}{r} \]

but apply it to a lens which has two radii of curvature.

The light refracting off the second surface is focussed at a distance \( i \) to the right of the surface, \( \cdots \) and behaves as an object at position \( p' = -i \) for the second surface.
\[ \frac{1}{p} + \frac{n}{i'} = \frac{n-1}{r_1} \]  

(1)

The image at \( i' \) behaves as a virtual object for the second surface, so in

\[ \frac{n}{p'} + \frac{1}{i} = \frac{1-n}{r_2} \Rightarrow -\frac{n}{i'} + \frac{1}{i} = \frac{1-n}{r_2} \]  

\( (p' = -i') \)

(1) & (2)

\[ \frac{1}{p} + \frac{1}{i} = (n-1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

Lensmaker equation

\[ \frac{1}{f} = (n-1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]
\[ |r_1| = |r_2| = 10\text{ cm} \]
\[ n = 1.52 \]

What is the focal length? 

- \( r_1 > 0 \) \( r_1 = +10\text{ cm} \)
- \( r_2 < 0 \) \( r_2 = -|r_2| = -10\text{ cm} \)

\[ \frac{1}{f} = (1.52 - 1) \left( \frac{1}{10\text{ cm}} - \frac{1}{-10\text{ cm}} \right) = 0.52 \times \frac{2}{10\text{ cm}} = \frac{0.52}{5\text{ cm}} \]

\[ \Rightarrow f = 9.6\text{ cm}. \]
34.4c. **Graphical Methods for Lenses.**

1. Parallel ray → \( F_2 \)
2. Ray through center
3. Ray through \( F_1 \) → parallel
\[ \frac{i}{p} \text{ bigger} \]

\[ \frac{i}{p} = \infty \]

image at infinity

image virtual
& larger than object
(magnifying glass)

Light rays converging on lens.
f = 20 cm

Calculate \( m \) & \( i \)

a) \( s = 50 \text{ cm} \)
\[
\frac{1}{s} + \frac{1}{i} = \frac{1}{f} \quad \Rightarrow \quad i = 33.3 \text{ cm} \quad m = \frac{-33.3}{50} = -0.66
\]

b) \( s = 20 \text{ cm} \)
\[
\frac{1}{s} + \frac{1}{i} = \frac{1}{f} \quad \Rightarrow \quad i = 50 \text{ cm} \quad m = 3
\]

c) \( s = 15 \text{ cm} \)
\[
\frac{1}{s} + \frac{1}{i} = \frac{1}{f} \quad \Rightarrow \quad i = 100 \text{ cm} \quad m = \frac{100}{15} = 6.67
\]

d) \( s = -40 \text{ cm} \)
\[
\frac{1}{s} + \frac{1}{i} = \frac{1}{f} \quad \Rightarrow \quad i = 75 \text{ cm} \quad m = \frac{-1}{40} \approx -0.025
\]

Image of an image.

\[ f = 8.0 \text{ cm} \quad f > 6.0 \text{ cm} \]

Find positive, size & orientation of image produced.

\[
\frac{1}{12} + \frac{1}{11} = \frac{1}{8.0} \quad \Rightarrow \quad i_1 = +24 \text{ cm} \quad \Rightarrow \quad p_2 = 12 \text{ cm}
\]

\[
\frac{1}{12} + \frac{1}{6} = \frac{1}{8.0} \quad \Rightarrow \quad i_2 = 12 \text{ cm} \quad \Rightarrow \quad m_2 = \frac{12}{12} = 1
\]

\[
m_{TOT} = m_1 m_2 = +2
\]
\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{f}
\]

Two lenses: image of second is object of first.

Example:
- \( p = 20 \text{ cm} \)
- \( f_1 = 10 \text{ cm} \)
- \( f_2 = 5 \text{ cm} \)
- \( d = 40 \text{ cm} \)

\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad \frac{1}{20} + \frac{1}{i} = \frac{1}{10} \quad \frac{1}{i} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \Rightarrow i = 20 \text{ cm}
\]

\[
m_1 = -1
\]

\[
p_2 = d - i = 40 - 20 = 20 \text{ cm}
\]

\[
\frac{1}{20} + \frac{1}{i'} = \frac{1}{5} \quad \frac{1}{i'} = \frac{1}{5} - \frac{1}{20} = \frac{4 - 1}{20} = \frac{3}{20}
\]

\[
i' = \frac{20}{3} = 6.6 \text{ cm}
\]

\[
m_2 = -\frac{1}{3} \quad m_1m_2 = +1 \Rightarrow m_1m_2 = \frac{-1}{3}
\]