2. IMAGES AND RAY OPTICS

Everything we see, everything, depends on ray optics. From our reflection in a mirror to images of distant galaxies caught by a space telescope— all depend on the way that light from a distant object is manipulated so that it appears to come from an image. Understanding the relationship between the positions, sizes & orientations of the object & image is the topic of lecture 2, and we shall make extensive use of ray optics.
0 — the location of a self-luminous point **OBJECT**

**O**

I — apparent origin of light rays

The **IMAGE** point

If rays do not actually pass through the image point **I**, it is said to be a **VIRTUAL IMAGE**.
Extended Objects

Represented by an upright arrow. Each point on the object acts as a point source. To locate the position of the image we draw some of the rays that reach the mirror from the top of the object.

\[ \Delta OVB = \Delta IVB \Rightarrow |p| = |i| \text{ for all } P \& I \]

Light from B appears as an image at B'.

But the light from B originates at O! Applying the laws of reflection we see an image of ourselves, nine sections away.
34.2 Spherical Mirrors

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} = \frac{2}{r} \]

\[ m = -\frac{i}{p} \]

Mirror Equation:
(Spherical mirror)

Lateral magnification

\[ \alpha + \delta = \phi \]
\[ \phi + \theta = \beta \]

Cube of opposite interior angles

\[ \Rightarrow \alpha + \beta = (\phi - \theta) + (\phi + \theta) = 2\phi \]

\[ \frac{h}{p-\delta} = \tan \alpha \]
\[ \frac{h}{r-\delta} = \tan \phi \]
\[ \frac{h}{i-\delta} = \tan \beta \]

\[ \alpha \sim \tan \alpha \quad \phi \sim \tan \phi \quad \beta \sim \tan \beta \]

\[ \alpha + \beta = \frac{h}{p} + \frac{h}{i} = \frac{2h}{r} \]

\[ \Rightarrow \frac{1}{p} + \frac{1}{i} = \frac{2}{r} \]
The mirror equation is an approximate formula, valid for small angles. All rays from 0 to 1

\[ \frac{1}{\infty} + \frac{1}{i} = \frac{2}{r} \implies i = \frac{r}{2} \]

But this is the focal length \( f = \frac{r}{2} \).

\[ \frac{1}{p} + \frac{1}{\infty} = \frac{2}{r} \quad p = \frac{r}{2} \]

Image at \( \infty \implies \) object at focal point

Object at \( \infty \implies \) image at focal point.

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad f = \frac{r}{2} \]
Sign Rules

Notice that if \( r = \infty \), we get the flat mirror equation \( p = -i \).

To clarify these minus signs we need a sign rule convention, which is as follows.

OBJECT

same side as

\( p > 0 \)

\( (p < 0 \text{ otherwise}) \)

same side as

\( i > 0 \)

\( (i < 0 \text{ otherwise}) \)

radius of
curvature

\( r > 0 \)

\( (r < 0 \text{ otherwise}) \)

same side as

\( \text{outgoing light} \)

\( \text{incoming light} \)
Image of a concave spherical mirror

\[ m = \frac{h'}{h} \]

\[ \text{OVA} \& \ IVQ' \text{ are similar} \Rightarrow \frac{h}{P} = -\frac{h'}{i} \quad (h' < 0) \]

\[ m = \frac{h'}{h} = -\frac{i}{P} \]

lateral magnification

If \( m < 0 \) then the image is inverted relative to the object.
Fish Stalker

Where does the stalker see the fish: a or b?

\[ \frac{\theta_2}{\theta_1} = \frac{n_2}{n_1} = 1.33 \]

Since \( n_1 = 1.33 > n_2 \), retroflexion occurs towards the normal.

a is the location of the image.
34.2 A moth at eye level is 10cm in front of a plane mirror; you are behind the moth, 30cm from the mirror. What is the distance from your eyes to the apparent position of the moth image?

34.6 An object is moved along the normal axis of a spherical mirror while the lateral magnification is measured.

What is \( m \) for \( p = 14 \text{cm} \)

\[
2 = \frac{f}{\theta + f - s} \quad \Rightarrow \quad 2(\theta - f) = f
\]

\[
\theta = \frac{10}{3} \quad \Rightarrow \quad f = 10
\]

\[
m(p=14) = \frac{10}{10-14} = \frac{10}{-4} = -2.5
\]
Convex mirrors \((r < 0)\)

\[
\begin{align*}
\beta &= \Theta + \phi \\
\Theta &= \phi + \alpha
\end{align*}
\]

\[
\alpha - \beta = (\Theta - \phi) - (\Theta + \phi) = -2\phi
\]

\[
\alpha = \frac{h}{p} \quad \beta = -\frac{h}{i} \quad \phi = -\frac{h}{r}
\]

\[
\Rightarrow (\alpha - \beta) = h \left( \frac{1}{p} + \frac{1}{i} \right) = \frac{2h}{r} \Rightarrow \frac{1}{p} + \frac{1}{i} = \frac{2}{r}
\]

excepting now \(r < 0\).

\[
\frac{1}{\infty} + \frac{1}{i} = \frac{2}{r} \Rightarrow i = \frac{r}{2} = -\frac{1}{2}
\]

\[
\frac{1}{p} + \frac{1}{\infty} = \frac{2}{r}
\]

\[
\Rightarrow p = \frac{r}{2} = -\frac{1}{2}
\]
Santa Claus reflected in an ornament

donament = 7.2 cm

\( h_{\text{Santa}} = 1.6 \text{ m}, \text{ standing } \ p = 0.75 \text{ m from the ornament } \)

How high is the image of Santa?

\[
\frac{1}{p} + \frac{1}{i} = \frac{-1}{1.8} \quad \frac{1}{i} = \frac{-1}{1.8} \cdot \frac{75}{75} \\
\Rightarrow i = -1.758 \text{ cm}
\]

\[
m = \frac{-i}{p} = \frac{-1.758}{75} = 0.0234
\]

\[
h' = h_m = 0.0234 \times 1.6 = 0.0374 \text{ cm} \quad \text{Virtual & upright}
\]
**Graphical Methods** (mirrors).

1. **Parallel ray** → Passes through F on reflection.

2. **Focal ray**, passing through F → Parallel on reflection.

3. **Radial ray** through center of curvature C, bounces back along radial path through C.

4. **Vertex ray**, passing through the vertex, is reflected forming equal angles with the opic axis.

PaFoRaVe
Real, inverted.

Virtual, upright.
\[ \frac{n_1 + n_2}{p} \cdot \frac{h}{i} = \frac{n_2 - n_1}{r} \]

Object-image relation
Spherical refracting Surface.

\[ \theta_1 = \alpha + \phi \\
\phi = \theta_2 + \beta \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \Rightarrow n_1 \theta_1 = n_2 \theta_2 \Rightarrow n_2 (\alpha + \phi) = n_2 (\phi - \beta) \]

\[ \tan \alpha = \frac{h}{p} \quad \tan \phi = \frac{h}{r} \quad \tan \beta = \frac{h}{i} \]

\[ \Rightarrow \quad \alpha = \frac{h}{p} \quad \phi = \frac{h}{r} \quad \beta = \frac{h}{i} \Rightarrow \quad n_1 \left( \frac{h + h}{p\cdot r} \right) = n_2 \left( \frac{h - h}{r\cdot i} \right) \]

Snell's law becomes

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ n_1 \left( \frac{h}{p} \right) = n_2 \left( \frac{h}{i} \right) \Rightarrow \quad m = \frac{h^{'}}{h} = -\left( \frac{i}{n_2} \right) \]
e.g. **Plane Surface** $(r = \infty)$

\[
\frac{n_1}{p} + \frac{n_2}{i} = 0
\]

\[\Rightarrow i = -p \left( \frac{n_2}{n_1} \right)\]

**Swimming pool** \(i = -p \left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) = -\frac{p}{1.33}\)

Image appears \(3/4\) of actual depth.