Spherical Mirror 1

Description: This problem requires students to determine the spherical mirror that would create a prescribed image.

You wish to create an image that is 10 meters from an object. This image is to be inverted and half the height of the object. You wish to accomplish this using one spherical mirror.

Part A

What is the focal length \( f \) of the mirror that would accomplish this?

Express your answer in meters, as a fraction or to three significant figures.

Hint 1. Find the focal length

There is a simple equation relating \( s_{ob} \) (the object distance) and \( s_{im} \) (the image distance) to \( f \). Use this equation to obtain \( f \) in terms of \( s_{ob} \) and \( s_{im} \).

\[
f = \frac{s_{ob}s_{im}}{s_{ob} + s_{im}}
\]

Express your answer in terms of \( s_{ob} \) and \( s_{im} \).

ANSWER:

If you are having trouble finding the values of \( s_{ob} \) and \( s_{im} \), Parts A.2 and A.3 should be helpful.

Hint 2. Find the magnification

The magnification \( m \) is defined as the height of the image \( h_{im} \) divided by the height of the object \( h_{ob} \). Find the magnification of the desired mirror.

Express your answer as a fraction.

ANSWER:

\[ m = -0.500 \]

Hint 3. Use the magnification to find \( s_{ob} \) and \( s_{im} \)

A second definition of magnification is \( m = \frac{s_{im}}{s_{ob}} \). Using the magnification you calculated in the Part A.2, find \( s_{ob} \) in terms of \( s_{im} \).

Express your answer in terms of \( s_{im} \).

ANSWER:

\[ s_{ob} = 2s_{im} \]
Now that you know that \( s_{oh} = 2s_{oh'} \), look again at the introduction to the problem. You should be able to find a second, simple equation relating \( s_{oh} \) and \( s_{oh'} \) based on the information in the introduction. Solve the two equations to obtain numerical values of \( s_{oh} \) and \( s_{oh'} \).

\[ f = 6.67 \text{ m} \]

Part B

Is this a concave or convex mirror?

ANSWER:

- concave
- convex

Remember that concave mirrors have positive focal lengths, and convex mirrors have negative focal lengths. You calculated a positive focal length in Part A, so the mirror must be concave.

Part C

What is the magnitude \( r \) of the radius of curvature of this mirror?

Express your answer in meters, as a fraction or to three significant figures.

**Hint 1. Relating focal length and radius of curvature**

For spherical mirrors, the focal length \( f \) is half of the radius of curvature \( r \) (i.e., \( f = r/2 \)).

ANSWER:

\[ r = 13.3 \text{ m} \]

Part D

What type of image is created, real or virtual?

**Hint 1. Real versus virtual**

A real image is one that forms at a location where the light actually reaches. (Imagine a slide projector creating an image on a screen.) A virtual image is one that forms at a location that the light does not actually reach. (Imagine the image created when you look in a mirror hanging on a wall. No light actually makes it past the wall. There only appears to be a copy of you on the other side of the wall!)

ANSWER:

- real
- virtual

Real images have positive image distance, and virtual images have negative image distance. Since you calculated a positive image distance in Part A, the image must be real.
Description: ± Includes Math Remediation. Qualitative and quantitative questions about image formation by lenses.

Learning Goal:
To learn the quantitative use of the lens equation, as well as how to determine qualitative properties of solutions.

In working with lenses, there are three important quantities to consider: The object distance \( s \) is the distance along the axis of the lens to the object. The image distance \( s' \) is the distance along the axis of the lens to the image. The focal length \( f \) is an intrinsic property of the lens. These three quantities are related through the equation

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

Note that this equation is valid only for thin, spherical lenses. Unless otherwise specified, a lens problem always assumes that you are using thin, spherical lenses.

The equation above allows you to calculate the locations of images and objects. Frequently, you will also be interested in the size of the image or object, particularly if you are considering a magnifying glass or microscope. The ratio of the size of an image to the size of the object is called the magnification. It is given by

\[
\frac{y'}{y} = \frac{s'}{s}
\]

where \( y' \) is the height of the image and \( y \) is the height of the object. The second equality allows you to find the size of the image (or object) with the information provided by the thin lens equation.

All of the quantities in the above equations can take both positive and negative values. Positive distances correspond to real images or objects, while negative distances correspond to virtual images or objects. Positive heights correspond to upright images or objects, while negative heights correspond to inverted images or objects. The following table summarizes these properties:

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>real</td>
</tr>
<tr>
<td>( s' )</td>
<td>real</td>
</tr>
<tr>
<td>( y )</td>
<td>upright</td>
</tr>
<tr>
<td>( y' )</td>
<td>upright</td>
</tr>
</tbody>
</table>

The focal length \( f \) can also be positive or negative. A positive focal length corresponds to a converging lens, while a negative focal length corresponds to a diverging lens.

Consider an object with \( s = 12 \text{ cm} \) that produces an image with \( s' = 15 \text{ cm} \). Note that whenever you are working with a physical object, the object distance will be positive (in multiple optics setups, you will encounter “objects” that are actually images, but that is not a possibility in this problem). A positive image distance means that the image is formed on the side of the lens from which the light emerges.

**Part A**
Find the focal length of the lens that produces the image described in the problem introduction using the thin lens equation.

Express your answer in centimeters, as a fraction or to three significant figures.

**ANSWER:**

\[ f = 6.67 \text{ cm} \]

**Part B**
Considering the sign of \( f \), is the lens converging or diverging?

**ANSWER:**

- converging
- diverging

**Part C**
What is the magnification \( m \) of the lens?

Express your answer as a fraction or to three significant figures.

**ANSWER:**

\[ m = -1.25 \]

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**Part D**

Think about the sign of \( y' \) and the sign of \( y \), which you can find from the magnification equation, knowing that a physical object is always considered upright. Which of the following describes the nature and orientation of the image?

**ANSWER:**

- real and upright
- real and inverted
- virtual and upright
- virtual and inverted

Now consider a diverging lens with focal length \( f = -15 \text{ cm} \), producing an upright image that is \( \frac{5}{9} \) as tall as the object.

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**Part E**

Is the image real or virtual? Think about the magnification and how it relates to the sign of \( y' \).

**ANSWER:**

- real
- virtual

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**Part F**

What is the object distance? You will need to use the magnification equation to find a relationship between \( s \) and \( y' \). Then substitute into the thin lens equation to solve for \( s \).

Express your answer in centimeters, as a fraction or to three significant figures.

**ANSWER:**

\[ s = 12.0 \text{ cm} \]

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**Part G**

What is the image distance?

Express your answer in centimeters, as a fraction or to three significant figures.

**ANSWER:**

\[ y' = -6.67 \text{ cm} \]

A lens placed at the origin with its axis pointing along the x axis produces a real inverted image at \( x = -24 \text{ cm} \) that is twice as tall as the object.

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**Part H**

What is the image distance?

Express your answer in centimeters, as a fraction or to three significant figures.
Part I
What is the x coordinate of the object? Keep in mind that a real image and a real object should be on opposite sides of the lens. Express your answer in centimeters, as a fraction or to three significant figures.
ANSWER:

\[ x = 12.0 \text{ cm} \]

Part J
Is the lens converging or diverging?
ANSWER:

○ converging
○ diverging

You can solve the lens equation for \( s' \) in terms of \( s \) and \( f \). If you do this and then substitute your result into the magnification equation, you will see that the only way to obtain an image of a real object that is larger than the object itself is with a converging lens.

Part K
Find the focal length of the lens. Express your answer in centimeters, as a fraction or to three significant figures.
ANSWER:

\[ f = 8.00 \text{ cm} \]

A Convex-Convex Lens
Description: Lensmaker's equation for a lens in air, then a lens in water.

A convex-convex lens has radii of curvature with magnitudes of \( |R_1| = 10 \text{ cm} \) and \( |R_2| = 15 \text{ cm} \). The lens is made of glass with index of refraction \( n_{\text{glass}} = 1.5 \). We will employ the convention that \( R_1 \) refers to the radius of curvature of the surface through which light will enter the lens, and \( R_2 \) refers to the radius of curvature of the surface from which light will exit the lens.

Part A
Is this lens converging or diverging?
ANSWER:

○ converging
○ diverging

Part B
What is the focal length \( f \) of this lens in air (index of refraction for air is \( n_{\text{air}} = 1 \))? Express your answer in centimeters to two significant figures or as a fraction.
Hint 1. The signs of the radii of curvature
The sign convention for the lensmaker's equation is that a radius of curvature is positive if the center of curvature is on the same side of the lens from which light would emerge and is negative otherwise.

Hint 2. The lensmaker's equation
The lensmaker's equation is \( \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \) where \( f \) is the focal length, \( n \) the refractive index of the lens material, and \( R_1 \) and \( R_2 \) the radii of curvature.

ANSWER:
\[ f = 12 \text{ cm} \]

Part C
What is the focal length of the lens if it is immersed in water (\( n_{\text{water}} = 1.3 \))?
Express your answer in centimeters, to two significant figures or as a fraction.

Hint 1. Putting the lens in water
You can account for a lens being in a medium other than air by replacing \( n \) in the lensmaker's equation with \( \frac{n_{\text{medium}}}{n_{\text{medium}}} \). Notice that if you let \( n_{\text{medium}} = 1 \), which is the case for air, you recover the original equation.

ANSWER:
\[ f = 39 \text{ cm} \]

Focusing with the Human Eye

Description: A man looks at three objects at different distances. Explain how he must adjust the lenses in his eyes to bring the various objects into focus.

Joe is hiking through the woods when he decides to stop and take in the view. He is particularly interested in three objects: a squirrel sitting on a rock next to him, a tree a few meters away, and a distant mountain. As Joe is taking in the view, he thinks back to what he learned in his physics class about how the human eye works.

Light enters the eye at the curved front surface of the cornea, passes through the lens, and then strikes the retina and fovea on the back of the eye. The cornea and lens together form a compound lens system. The large difference between the index of refraction of air and that of the aqueous humor behind the cornea is responsible for most of the bending of the light rays that enter the eye, but it is the lens that allows our eyes to focus. The ciliary muscles surrounding the lens can be expanded and contracted to change the curvature of the lens, which in turn changes the effective focal length of the cornea-lens system. This in turn changes the location of the image of any object in one's field of view. Images formed on the fovea appear in focus. Images formed between the lens and the fovea appear blurry, as do images formed behind the fovea. Therefore, to focus on some object, you adjust your ciliary muscles until the image of that object is located on the fovea.

Part A
Joe first focuses his attention (and his eyes) on the tree. The focal length of the cornea-lens system in his eye must be __________ the distance between the front and back of his eye.
Part B

Joe's eyes are focused on the tree, so the squirrel and the mountain appear out of focus. This is because the image of the squirrel is formed ______ and the image of the mountain is formed ______.

**Hint 1. Image of the squirrel**

The squirrel is closer to the lens (the eye) than the tree. As long as Joe's eyes stay fixed on the tree, their focal length does not change. Using the lens equation, determine whether the image of the squirrel is closer to the lens than the image of the tree or farther away.

**Hint 2. Image of the mountain**

The mountain is farther from the lens (the eye) than the tree. As long as Joe's eyes stay fixed on the tree, their focal length does not change. Using the lens equation, determine whether the image of the mountain is closer to the lens than the image of the tree or farther away.

**ANSWER:**

- between the lens and fovea / between the lens and fovea
- between the lens and fovea / behind the fovea
- behind the fovea / between the lens and fovea
- behind the fovea / behind the fovea

Part C

Joe now shifts his focus from the tree to the squirrel. To do this, the ciliary muscles in his eyes must have _____ the curvature of the lens, resulting in a(n) ______ focal length for the cornea-lens system. Note that curvature is different from radius of curvature.

**ANSWER:**

- increased / increased
- increased / decreased
- decreased / increased
- decreased / decreased

Part D

Finally, Joe turns his attention to the mountain in the distance but finds that he cannot bring the mountain into focus. This is because he is nearsighted. But when Joe puts on his glasses, he can see the mountain clearly. To adjust for his nearsightedness, his glasses must contain _____ lenses.
Hint 1. Focusing on distant objects
The image of a distant object like the mountain always forms at (or very close to) the focal point of the fovea-lens system. When Joe is looking at the most distant object he can see clearly, where is the focal point?

Hint 2. The role of corrective lenses
Nearsightedness and farsightedness are both caused by the fact that the ciliary muscles cannot make the focal length of the lens arbitrarily large or small. The corrective lenses must make the image of the distant mountains form someplace that his eyes are naturally able to focus on.

ANSWER:
- converging
- diverging

Constructive and Destructive Interference Conceptual Question
Description: Conceptual question on whether constructive or destructive interference occurs at various points between two wave sources.

Two sources of coherent radio waves broadcasting in phase are located as shown below. Each grid square is 0.5 m square, and the radio sources broadcast at \( \lambda = 2.0 \text{ m} \).

Part A
At Point A is the interference between the two sources constructive or destructive?

Hint 1. Path-length difference
Since the two sources emit radio waves in phase, the only possible phase difference between the waves at various points is due to the different distances the waves have traveled to reach those points. The difference in the distances traveled by the two waves from source to point of interest is termed the path-length difference.

If the path-length difference is an integer multiple of the wavelength of the waves, one wave will pass through an integer number of complete cycles more than the other wave, placing the two waves back in perfect synchronization, resulting in constructive interference. If the path-length difference is a half-integer multiple of the wavelength, one wave will be one-half of a cycle, or 180 degrees, out of phase, resulting in destructive interference.

Hint 2. Find the path-length difference
What is the distance from the left source to Point A, \( d_{A,\text{left}} \)? What is the distance from the right source to Point A, \( d_{A,\text{right}} \)?

Enter the distances in meters separated by a comma.

ANSWER:
\[ d_{A,\text{left}}, d_{A,\text{right}} = 3.3 \text{ m}, 3.3 \text{ m} \]
The wave that leaves the source on the left and the wave that leave the source on the right travel equal distances to Point A.

**Part B**

At Point B is the interference between the two sources constructive or destructive?

**Hint 1.** Find the path-length difference

What is the distance from the left source to Point B, \( d_{\text{left}} \)? What is the distance from the right source to Point B, \( d_{\text{right}} \)?

Enter the distances in meters separated by a comma.

**ANSWER:**

\[ d_{\text{left}}, d_{\text{right}} = 1.5, 4.5 \text{ m} \]

The path-length difference between the two waves is 3 m.

**ANSWER:**

- constructive
- destructive

**Part C**

At Point C is the interference between the two sources constructive or destructive?

**ANSWER:**

- constructive
- destructive

**Part D**

At Point D is the interference between the two sources constructive or destructive?

**ANSWER:**

- constructive
- destructive

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**Exercise 35.5**

**Description:** Two speakers, emitting identical sound waves of wavelength 2.0 m in phase with each other, and an observer are located as shown in the figure. (a) At the observer’s location, what is the path difference for waves from the two speakers? (b) Will the ...

Two speakers, emitting identical sound waves of wavelength 2.0 m in phase with each other, and an observer are located as shown in the figure.
Part A
At the observer's location, what is the path difference for waves from the two speakers?

Express your answer using two significant figures.

ANSWER:

\[ \Delta P = 2.0 \text{ m} \]

Part B
Will the sound waves interfere constructively or destructively at the observer's location-or something in between constructive and destructive?

ANSWER:

- destructive
- constructive
- something in between constructive and destructive

Part C
Suppose the observer now increases her distance from the speakers to 17.0 m, staying directly in front of the same speaker as initially. At the observer's new location, what is the path difference for waves from the two speakers?

Express your answer using two significant figures.

ANSWER:

\[ \Delta P = 1.0 \text{ m} \]

Part D
Will the sound waves interfere constructively or destructively at the observer's location-or something in between constructive and destructive?

ANSWER:

- destructive
- constructive
- something in between constructive and destructive

Multiple Choice Question - 35.2
Description: (a) Two radio antennas are 100 m apart on a north-south line. The two antennas radiate in phase at a frequency of 4.9 MHz. All radio measurements are made far from the antennas. The smallest angle, reckoned north of east from the antennas, at which ...
Part A

Two radio antennas are 100 m apart on a north-south line. The two antennas radiate in phase at a frequency of 4.9 MHz. All radio measurements are made far from the antennas. The smallest angle, reckoned north of east from the antennas, at which destructive interference of the two radio waves occurs, is closest to:

ANSWER:

- 18°
- 22°
- 8.9°
- 27°
- 13°

Double Slit 1

Description: This problem explores double-slit interference maxima and minima.

Two lasers are shining on a double slit, with slit separation \( d \). Laser 1 has a wavelength of \( \lambda_1 \), whereas laser 2 has a wavelength of \( \lambda_2 \). The lasers produce separate interference patterns on a screen a distance 5.40 m away from the slits.

Part A

Which laser has its first maximum closer to the central maximum?

**Hint 1. Path difference**

The first maximum comes when the path difference between the two slits is equal to one full wavelength. Think about which laser has a smaller wavelength, and recall that the distance from the central maximum is proportional to the path difference.

ANSWER:

- laser 1
- laser 2

Part B

What is the distance \( \Delta y_{\text{max-max}} \) between the first maxima (on the same side of the central maximum) of the two patterns?

Express your answer in meters.

**Hint 1. Find the location of the first maximum for laser 1**

The first maximum corresponds to constructive interference, with \( m = 1 \) (since the central maximum corresponds to \( m = 0 \)). Using the small-angle approximation, what is the distance \( y_1 \) of this maximum from the central maximum for laser 1?

**Hint 1. Angle to maxima**

The angle to the \( m \)th maximum is given by \( d \sin(\theta) = m\lambda \), where \( \theta \) is the angle, \( d \) is the separation between the slits, and \( \lambda \) is the wavelength of the light.

**Hint 2. Distance on screen**

For a screen that is far from the slits, as in this problem, the distance \( y \) on the screen from the central maximum is \( y = R \sin(\theta') \), where \( \theta' \) is the angle from the slits to the point on the screen and \( R \) is the distance from the slits to the screen.
**Hint 2. Find the location of the first maximum for laser 2**

Now that you have found the first maximum for laser 1, what is the location $y_2$ of the first maximum for laser 2?

Express your answer in meters.

**Hint 1. Angle to maxima**

The angle to the $m$th maximum is given by $m \sin(\theta) = m \lambda$, where $\theta$ is the angle, $d$ is the separation between the slits, and $\lambda$ is the wavelength of the light.

**Hint 2. Distance on screen**

For a screen that is far from the slits, as in this problem, the distance $y$ on the screen from the central maximum is $y = R \sin(\theta)$, where $\theta$ is the angle from the slits to the point on the screen and $R$ is the distance from the slits to the screen.

**ANSWER:**

$$y_2 = \frac{L}{10} = 0.360 \text{ m}$$

To calculate the distance between these two maxima, just calculate the difference of the two distances.

**ANSWER:**

$$\Delta y_{\text{max-min}} = \frac{L}{60} = 0.00 \times 10^{-2} \text{ m}$$

Also accepted: $\frac{L}{59.7} = 9.05 \times 10^{-2}$

**Part C**

What is the distance $\Delta y_{\text{max-min}}$ between the second maximum of laser 1 and the third minimum of laser 2, on the same side of the central maximum?

Express your answer in meters.

**Hint 1. Find the location of the second maximum**

If the central maximum corresponds to $m_1 = 0$, then you should be able to figure out what the second maximum corresponds to. Using that, what is the distance $y_3$ to the second maximum of laser 1 from the central maximum?

Express your answer in meters.

**Hint 1. Angle to maxima**

The angle to the $m$th maximum is given by $m \sin(\theta) = m \lambda$, where $\theta$ is the angle, $d$ is the separation between the slits, and $\lambda$ is the wavelength of the light.

**Hint 2. Distance on screen**

For a screen that is far from the slits, as in this problem, the distance $y$ on the screen from the central maximum is $y = R \sin(\theta)$, where $\theta$ is the angle from the slits to the point on the screen and $R$ is the distance from the slits to the screen.
**Hint 2.** Find the value of the third minimum

The first minimum corresponds to \( m_1 = 0 \) (since there is no central minimum). What, then, is the value of \( m \) for the third minimum? Recall that \( m \) is always an integer.

**Express your answer as a whole number.**

**ANSWER:**

\[
m_{\text{third minimum}} = 2
\]

**Hint 3.** Find the location of the third minimum

Given that \( m_1 = 2 \), what is the location of the third minimum?

**Express your answer in meters.**

**Hint 1.** Difference between the angle to a maximum and a minimum

Once you have the value of \( m \), the equation for the angle to a minimum is almost identical to the equation for the angle to a maximum. The only difference is that you use \( m + 1/2 \) in place of \( m \) if you want to find the location of a minimum instead of a maximum.

**ANSWER:**

\[
\theta_{\text{third minimum}} = \frac{L}{6} = 0.900 \text{ m}
\]

Also accepted: \( \frac{L}{0.92} = 0.912 \)

\[
\Delta_{\text{max-min}} = \frac{L}{10} = 0.360 \text{ m}
\]

Also accepted: \( \frac{L}{14.5} = 0.370 \)