Physics 228

Today: More Diffraction/Interference

- Multiple slits interference
- Diffraction intensity algebra
Multiple Slits Phase Difference

\[ d = \text{slit separation} \]

\[ \Delta = d \sin \theta = \text{path length difference between adjacent slits} \]

\[ \phi = \frac{2\pi \Delta}{\lambda} = \frac{2\pi d}{\lambda} \sin \theta = \text{phase difference between adjacent slits} \]
Let’s add multiple waves using the phasor picture:

- This is useful for adding up two or more oscillations of arbitrary phases.
- Just add the corresponding phasors (as vectors).
- Then let the resultant phasor spin around the origin.
- The oscillation produced by the sum phasor is the sum of the original oscillations!
At a particular point in space, the electric field from two sources add according to the phasor diagram at right. Which of the following statements is true for the resulting intensity?

A) The resulting intensity is twice that of a single source
B) The resulting intensity is greater than if only one source was present
C) The resulting intensity is zero
D) The resulting intensity is smaller than if only one source was present
Multiple Slits Phasor Diagrams

- 14 slits
- look at center of screen
- $\varphi = 0$
- constructive interference

- 4 slits
- look at point not at center of screen
- $\varphi > 0$
- interference: neither minimum nor a maximum
N = 6 Slits Phasor Diagram
N = 3 Slits Phasor Diagrams

\[ E_r = 3E_0 \]
\[ \phi = 0 \]
\[ \delta = 0 \]
\[ I = 9I_0 \]

\[ E_r = 2E_0 \]
\[ \phi = 60^\circ = \pi/3 \]
\[ \delta = \lambda/6 \]
\[ I = 4I_0 \]

\[ E_r = 0 \]
\[ \phi = 120^\circ = 2\pi/3 \]
\[ \delta = \lambda/3 \]
\[ I = 0 \]

\[ E = E_0 \]
\[ \phi = 180^\circ = \pi \]
\[ \delta = \lambda/2 \]
\[ I = I_0 \]
N = 8 Slits Phasor Diagrams

(a) Phasor diagram for $\phi = \pi$

(b) Phasor diagram for $\phi = \frac{\pi}{4}$

(c) Phasor diagram for $\phi = \frac{\pi}{2}$

Above: phasor diagrams for three phase angles leading to destructive interference ($I = 0$)!
Clicker Question

Coherent light passing through six (6) slits separated by a distance \( d \) produces a pattern of dark and bright areas on a distant screen. There will be a dark area on the screen at a position where the path difference to the screen from adjacent slits is

A. \( \frac{\lambda}{2} \)

C. \( \frac{\lambda}{6} \)

D. any of these.
Multiple Slits Intensity Patterns

Graph showing intensity patterns for different numbers of slits (N = 2, 3, 4, 5, 10) with corresponding intensity values and distances.

Image of intensity patterns for different numbers of slits (w=50μ, d=150μ, 3 slits, 4 slits, 5 slits, 7 slits) with observed intensity levels.
For an N-slit interference pattern, how many minima / secondary maxima are there between the big peaks?

a) 0 minima, 0 maxima
b) 1 minimum, 0 maxima
c) N minima, N-1 maxima
d) N-1 minima, N maxima
e) N-1 minima, N-2 maxima
What happens for $N \to \infty$?

Distinguish two cases:

1. $N \to \infty$ and $d \to 0$, with $N \, d = a = \text{const.}$: Single Slit Diffraction

   $E = E_0 \frac{\sin(\beta/2)}{\beta/2}$

   $I = I_0 \left( \frac{\sin(\beta/2)}{\beta/2} \right)^2$

2. $N \to \infty$ with $d = \text{constant}$: Diffraction Grating

   **Simulation**
Single Slit: Width of Central Peak

(a) $a = \lambda$

If the slit width is equal to or narrower than the wavelength, only one broad maximum forms.

(b) $a = 5\lambda$

The wider the slit (or the shorter the wavelength), the narrower and sharper is the central peak.

(c) $a = 8\lambda$

Width of central peak $\approx$ angle of first minimum

$\theta \approx \sin \theta = \lambda/a$
For a single slit, what is the intensity at a point on the screen where the phase difference between the extremal paths (upper edge vs. lower edge of slit) is \(66 \text{ rad} = 10.5 \times 2\pi \text{ rad}^2\)?

a) Use \(I = I_0 \left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2\) where \(\beta\) is 66.

b) Use \(I = I_0 \left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2\) where \(\beta/2\) is 66.

c) Use \(I = I_0 \left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2\) where \(\beta\) is 10.5.

d) Use \(I = I_0 \left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2\) where \(\beta\) is the slit width/wavelength.

e) The problem cannot be solved without being given a and \(\lambda\).
For a diffraction grating, as demonstrated during the last lecture, the maxima for the different colors appear at different angles. (Maxima at $\sin \theta = m\lambda/d$.)

Consider looking at light in the region just off the central maxima. Moving away, the various color maxima will appear, going from shortest (blue) to longest (red) wavelength. This is called the “spectrum”.

Looking at stars, for example, one sees interesting features in the spectrum, which tell us a great deal about the atomic composition, the temperature, the velocity (w/r to us), even the speed at which the star rotates!
One measure of the quality of the spectrograph is its “chromatic resolving power” $R = \frac{\lambda}{\Delta \lambda}$, where $\Delta \lambda$ is the smallest separation between spectral features that can be resolved.
Diffraction-Grating Resolving Power

To be separated, the peaks for different features should not overlap.

\[ R = \frac{\lambda}{\Delta \lambda} = \frac{\varphi}{\Delta \varphi} \]

\[ \varphi = 2\pi m \text{ (for diffraction order } m) \]

\[ \Delta \varphi = \frac{2\pi}{N} \]

To separate, the peaks for different features should not overlap.

Width of peak: \( \Delta \varphi = \frac{2\pi}{N} \)

2\pi phase difference between adjacent slits