Today: Interferometry, Diffraction

Diffraction is a further expansion of the idea of interference: Instead of two sources we consider a large number of sources. Here is an example from around home:

CDs have grooves separated by about 1.6 μm. When light strikes the disk, each groove acts as a source, generating an interference pattern.
Interferometers

- Interferometers measure changes in optical path length to high precision.
- The type shown here was used by Michelson & Morley in 1887 to show that the speed of light does not depend on whether the light path is along the earth’s motion around the sun, or perpendicular to it.
- The highest precision Michelson-Morley interferometer is LIGO, used to search for ripples in space-time (gravitational waves). It’s “legs” are 4 km long.
Are Shadows Infinitely Sharp?

Geometric optics predicts that this situation should produce a sharp boundary between illumination and solid shadow.

That’s NOT what really happens!

Point source

Area of illumination

Geometric shadow

Screen
Are Shadows Infinitely Sharp?  

No.  

Many examples:

- PhET simulation, physical demo

With a thin slit, you can see the light expand to both sides and form light and dark bands.
Why Does Light “Spill” Past the Geometrical Edge?

Huygen’s Principle

Each point on the wave front can be viewed as a source for spherical waves propagating out from that point. The wave front is the envelope of all the secondary waves.

The sum of all these wavefronts will bend around corners.

(a) A slit as a source of wavelets

Slit width $a$

Each strip is a source of Huygens’s wavelets.

Plane waves incident on the slit
Huygen's Principle

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Diffraction = interference with many sources, not just a few.
• **Consider two situations:** One where the image is close to the slit, the other where the image is far from the slit.

• **We will stick to the second, simpler (Fraunhofer) case.**

• **Here we are far enough away that the rays that come to a point on the screen can be treated as parallel.**
**Single Slit Diffraction**

Idea: Mentally divide the slit into pairs of points, separated by \( \frac{a}{2} \). Require the paths through each pair of points to interfere destructively. This is the case of the path length difference is \( \frac{\lambda}{2} \).

For the two strips shown, the path difference to \( P \) is \( (\frac{a}{2}) \sin \theta \). When \( (\frac{a}{2}) \sin \theta = \frac{1}{2} \), the light cancels at \( P \). This is true for the whole slit, so \( P \) represents a dark fringe.

(b) Enlarged view of the top half of the slit

\( \theta \) is usually very small, so we can use the approximations \( \sin \theta = \theta \) and \( \tan \theta = \theta \). Then the condition for a dark band is

\[
y_m = x \frac{m\lambda}{a}
\]

**Condition for this destructive interference:**

\[
\frac{\lambda}{2} = (\frac{a}{2}) \sin \theta
\]

\( \Rightarrow \) \( \sin \theta = \frac{\lambda}{a} \)
Single Slit Diffraction

Are there other minima?

- Divide the slit into $2m$ sections ($m > 0$).
- Require section $i$ to interfere destructively with section $i+1$.
- Condition for destructive interference:
  \[ \lambda/2 = (a/2m) \sin \theta \]
- Get minima for \( \sin \theta = m\lambda/a \), with $m = \pm 1, \pm 2, \ldots$
- This argument does not work for $m = 0$!
Single Slit Diffraction: Intensity Distribution

At $m = 0$, there is a very bright central peak surrounded by much dimmer side peaks.

In the central peak ($m = 0$), all of Huygen’s wavelets are exactly in phase.
A single slit diffraction pattern with 500 nm light has a series of dark bands separated by 1 cm at a distance of 5 m from the slit. What is the width of the slit?

\[
\sin \theta = \frac{m \lambda}{a} \quad \text{or} \quad y = \frac{m R \lambda}{a}
\]

solve for \( a \):

\[
a = \frac{R \lambda}{y} \quad (\text{set } m = 1)
\]

\[
a = \frac{(5)(500 \times 10^{-9})}{0.01} \text{ m} = 0.25 \text{ mm}.
\]
Two Slits of Finite Width

- Single slit with width $a$:
  - Minima at $\sin \theta = m \lambda / a$ ($m = \pm 1, \pm 2, \ldots$)
  - Maxima at $m = 0$ and $m$ half-integer.

- Two slits, with separation $d$:
  - Maxima at $\sin \theta = m \lambda / d$ ($m = 0, \pm 1, \pm 2, \ldots$)
  - Minima at $m$ half-integer.

- Two slits are actually composed of two single slits with separation greater than their width.
- The intensity patterns of single and double slits combine:
Instead of two slits, consider many slits: “transmission grating”. Diffraction gratings are usually labelled by the number of slits per unit length.

A “reflection grating” is composed of reflecting grooves instead of slits.

The path length difference between adjacent slits is: $d\sin\theta$

There will be constructive interference between ALL slits when there is constructive interference between adjacent slits:

$$m\lambda = d\sin\theta \quad \text{so} \quad \sin\theta = \frac{m\lambda}{d}$$

This is the same algebra as for two-slit interference.

With the larger number of slits, the pattern gets sharper.

Let’s see a demo.
Reflection grating

Diffraction gratings allow us to separate the colors (wavelengths) of light: Construct a “monochromator”.