Today: More Interference

Today we expand on the concept of interference.

If two “sources” of radiation have a fixed phase relationship, depending on the path length to the point of observation, they can interfere constructively or destructively.

The energy in a wave depends on the amplitude (E-field, B-field) squared, and when you have multiple waves, just add the E- and B-fields. In general, you are adding vectors.
Two point sources oscillate in phase. Point P is 7.3 wavelengths from the first source, and 4.6 wavelengths from the second source. As a result, at point P there is

a. Constructive interference.
b. Destructive interference.
c. Interference, but somewhere between constructive and destructive.
d. No interference at all
e. Not enough information to decide.
Suppose Young’s double-slit experiment is performed in air using red light and then the apparatus is immersed in water. What happens to the interference pattern on the screen?

A. It disappears.
B. The bright fringes are closer together.
C. The color shifts toward blue.
D. The bright fringes are farther apart.
E. No change happens in the interference pattern.
At any given point in space, a wave is just a cosine oscillation.

We can visualize such an oscillation as the x-axis projection of a vector that rotates around the origin at angular frequency $\omega = \frac{2\pi}{T}$.

This rotating vector is called a "phasor", short for "phase vector".
This is useful for adding up two or more oscillations of arbitrary phases.

Just add the corresponding phasors (as vectors).

Then let the resultant phasor spin around the origin.

The oscillation produced by the sum phasor is the sum of the original oscillations!
Light Intensity = Light Energy / (Area x Time)

Intensity \( \propto (Electric \ Field \ Amplitude)^2 \)

Phasor picture:

\[
E_1(t) = E \cos(\omega t)
\]

\[
E_2(t) = E \cos(\omega t + \Phi)
\]

Interference:

\[
E_1(t) + E_2(t) = E_p \cos(\omega t + \theta)
\]

with \( E_p^2 = E^2 + E^2 + 2EE \cos(\Phi) \)

\( = 2E^2 [1 + \cos(\Phi)] \)

In particular:

\( E_p^2 = 4E^2 \) for \( \Phi = 0 \) (constructive int.)

\( E_p^2 = 0 \) for \( \Phi = 180^\circ \) (destructive int.)
2-Slit Interference Pattern

(c) Approximate geometry

Resultant amplitude:

\[ E_p^2 = 2 E^2 [1 + \cos(\Phi)] \]
\[ = 4 E^2 \cos^2(\Phi/2) \]

Intensity:

\[ I = I_0 \cos^2(\Phi/2). \]

... so we can treat the rays as parallel, in which case the path-length difference is simply \( r_2 - r_1 = d \sin \theta \).

Intensities maxima occur where \( \phi \) is an integral multiple of \( 2\pi \) and \( d \sin \theta \) is an integral multiple of \( \lambda \).
Interference in Thin Films

It looks like if the path lengths are different by \( m \) (\( m \) integer) wavelengths, we have constructive interference, but if the difference is \( (m + \frac{1}{2}) \) wavelengths, there is destructive interference.

Note: Recall that the wavelength of light in a medium is \( \lambda_{\text{medium}} = \lambda_{\text{vacuum}}/n \), and you need to use \( \lambda_{\text{medium}} \) when determining the path length difference. We will just call it \( \lambda \).
(a) Interference between rays reflected from the two surfaces of a thin film

Light reflected from the upper and lower surfaces of the film comes together in the eye at $P$ and undergoes interference.

Some colors interfere constructively and others destructively, creating the color bands we see.

(b) The rainbow fringes of an oil slick on water

BUT:

We need to consider possible phase changes when the light refracts into a medium or reflects from a surface!
Phase Changes

Let's repeat an old simulation and recall what happens with traveling waves on ropes, from way back in Physics 124...

The transmitted wave has the same phase as the incident wave. The reflected wave has either the same phase, or the opposite phase, depending on whether reflection occurs from a less dense or a denser medium, respectively.
The situation with light is similar to the situation with the rope. The transmitted / refracted wave has the same phase as the incident wave. The reflected wave has the same phase going from a higher n to lower n medium, but the opposite phase when going from a lower n to higher n medium.
Why the Phase Changes?

This is a topic for Physics 305: Modern Optics, or an E&M course.

The general idea is the following:

- According to Maxwell’s equations, the parallel component of the total electric field on the incident side (incident + reflected fields) must be the same as the total field (refracted) on the other side.
- The same is true for the perpendicular component of the magnetic field.
- From these “boundary conditions”, the amplitudes and phases of the reflected and refracted waves follow. (“Fresnel equations”)
- Note on the side: For total internal reflection, the phase change is non-trivial and depends on angle of incidence.
Reflected Field for Normal Incidence

For light normally incident from medium a to medium b:

\[ E_r = \frac{n_a - n_b}{n_a + n_b} E_i \]

If the light is incident from the larger n medium:
\( E_r \) is the same sign as \( E_i \) (no phase change).

If the light is incident from the smaller n medium:
\( E_r \) is the opposite sign as \( E_i \) (phase change).

The more similar the indices are, the smaller the numerator and the less light gets reflected. When the n’s are the same no light is reflected (remember the invisible beaker?)
Returning to the Oil Film

Oil has an index of refraction similar to glass, \( n \approx 1.50 \).
Water has \( n \approx 1.33 \).
Air has \( n = 1 \).

- There is a 180° phase shift when the light reflects from the air-oil boundary.
- There is no phase shift when light reflects from the oil-water boundary.
- For normal incidence, because of the added phase shift, we get destructive interference when \( 2t = m \lambda_{\text{oil}} \), and constructive interference when \( 2t = (m + \frac{1}{2}) \lambda_{\text{oil}} \).
When the oil gets very thin, what color do you see?

a) White! All the colors interfere constructively!
b) Black! All the colors interfere destructively!
c) Long wavelength is more constructive - red.
d) Short wavelength is more constructive - blue.
e) It is the medium wavelengths that dominate - yellow/green.
Demo!

Let’s look at an actual soap film.

We use a converging lens to project a real, inverted image of a thin soap film onto a screen.

The soap will drain from the top (bottom of the image) towards to bottom (top of the image). You should be able to see the colors of the spectrum. Why?
A Thin Air Film Between Glass

- Light reflects from the glass-air interface, with no phase change.
- Second reflection from the air-glass interface below, with a phase change.
- If the angle of incidence is small we get destructive interference when $2t = m\lambda$, and constructive interference when $2t = (m + \frac{1}{2})\lambda$.
- The phase change is at the bottom rather than the top surface, but the result is the same as for oil on water, or soap film in air.
Were you happy with that explanation?

But why didn’t we worry about the light reflecting from the top surface of the glass?

a) Uhhh.... I didn’t think of that.
b) Light does not reflect from the top of the top glass.
c) The light reflection at the air-glass interface at the top is always in phase with the air-glass interface at the bottom.
d) The glass is constant thickness so there is no interference.
e) The glass is thicker than $\frac{1}{2}$ the coherence length.
• In the picture above, the length of the light “bursts” represents the coherence length.

• For the thin film, the coherence length is large enough to allow interference.

• For the thick plate, the path length difference exceeds the coherence length, thus no interference.
Anti-Reflection Coatings

• An important application is the design of anti-reflection coatings for optical elements.

• Consider a thin film of thickness $\lambda/4$, and an index of refraction $1 < n_{\text{coating}} < n_{\text{glass}}$.

• When light reflects from the top vs bottom of the coating, there is a $\lambda/2$ path length difference.

• Thus there is destructive interference for the reflected wave.

• $n_{\text{coating}}$ may be chosen such that all the light is transmitted.

• Since the wavelengths of visible light vary by nearly a factor of 2 this does not work equally well for all wavelengths at the same time.
Newton’s Rings

(a) A convex lens in contact with a glass plane

(b) Newton’s rings: circular interference fringes

- Thin film of air between a lens and a plane glass.
- Produces circular interference fringes.
- Because of the curvature, the intensity oscillates more rapidly near the edge than near the center.
- Dark spot in the center. Why?
Application of Newton’s Rings

Fringes map lack of fit between lens and master.

The interference pattern allows the shape / uniformity of a lens to be tested, compared to a “master”.