Physics 228

Today: Interference

www.physics.rutgers.edu/ugrad/228

You have studied interference at some level in 124, so this should be in some respects review.
Clicker Question

wavelength x frequency = speed of light

\[ \lambda f = c \]

In a vacuum, red light has a wavelength of 700 nm and violet light has a wavelength of 400 nm. This means that in a vacuum, red light

A) has higher frequency and moves faster than violet light.
B) has higher frequency and moves slower than violet light.
C) has lower frequency and moves faster than violet light.
D) has lower frequency and moves slower than violet light.
E) none of the above
Interference

Why do we see the colors in the soap bubble?

Interference.

We will begin to see how this works today, looking at interference of two sources.

(Soap bubbles will be considered on Thursday.)
Electromagnetic Waves

Maxwell’s equations:

\[ \nabla \cdot \mathbf{E} = 0 \]  
(no charges)

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]  
(no currents)

Plane wave solutions:

\[ \mathbf{E} = E_0 \cos(kz - \omega t) \hat{x} \]

\[ \mathbf{B} = B_0 \cos(kz - \omega t) \hat{y} \]

with propagation speed

\[ c = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 2.9979 \times 10^8 \text{ m/s} \]
Clicker Question

Which equation describes a wave moving in the negative x direction?

(a) \( y(x,t) = A \cos(kx - \omega t) \)

(b) \( y(x,t) = A \cos(-kx - \omega t) \)

(c) \( y(x,t) = A \cos(kx + \omega t) \)

(d) \( y(x,t) = -A \cos(kx - \omega t) \)

(e) both (b) and (c)
Waves From Point Sources

Sound Waves
(scalar field, or longitudinal vector field)

Wave fronts: crests of the wave (frequency $f$) separated by one wavelength $\lambda$

The wave fronts move outward from source $S_1$ at the wave speed $v = f\lambda$.

Spherical Wave

EM Waves
(transverse vector field)

Dipole radiation
(drawing by Heinrich Hertz)
The two sources emit waves “in phase” - crests and troughs are emitted at the same time from each source.

The waves are also “in phase” at points a and b, but “out of phase” at point c.

The distances from the sources to a are the same.

The distances from the sources to b are different by 2 wavelengths.

The distances to point c are different by 2.5 wavelengths.
Constructive and Destructive Interference

(b) Conditions for constructive interference: Waves interfere constructively if their path lengths differ by an integral number of wavelengths: \( r_2 - r_1 = m\lambda \).

(c) Conditions for destructive interference: Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths: \( r_2 - r_1 = (m + \frac{1}{2})\lambda \).

- At points where the waves add up “in phase”: constructive interference (maxima)
- At points where the waves add up “out of phase”: destructive interference (minima)
- All other places are somewhere in between these limits.
Let's see a demo and a simulation of 2-source wave interference.

I am going to move the sources apart. Will the minima/maxima move?

a) No! Of course not.
b) Yes! The angles between them increase and you get fewer of them.
c) Yes! The angles between them decrease and you get more of them.
d) Yes! You get more minima lines but fewer maxima lines.
e) Yes! You get more maxima lines but fewer minima lines.
I am going to change the light from red to blue (make the wavelength shorter.) What happens to the interference pattern?

a) Nothing changes, except the pattern becomes blue.

b) The maxima move in and there are more of them.

c) The maxima move out and there are fewer of them.

d) The angles decrease, but there are still the same number minima / maxima lines.

e) None of the above
iClicker:

Two point sources oscillate in phase. Point P is 7.3 wavelengths from the first source, and 4.3 wavelengths from the second source. As a result, at point P there is

a. Constructive interference.
b. Destructive interference.
c. Interference, but somewhere between constructive and destructive.
d. No interference at all
e. Not enough information to decide.
Maxima and Minima

Assume that two sources are in phase.

For maxima, we want constructive interference:
\[ r_2 - r_1 = m\lambda, \text{ with } m = 0, \pm 1, \pm 2, \ldots \]

For minima, we want destructive interference:
\[ r_2 - r_1 = (m+\frac{1}{2})\lambda, \text{ with } m = 0, \pm 1, \pm 2, \ldots \]
Coherence

Coherent wave

Incoherent waves
Coherence

Two types of coherence:

• **Temporal/spectral coherence** (Only one frequency / a range of frequencies)

• **Spatial coherence** (all waves travel in the same direction / in a range of directions)

Natural light (e.g. sunlight) is incoherent. This is the main reason interference is not often seen in everyday life.
Create Coherent Light

To create **spectral coherence**: Pass through a “monochromator” (color filter) which passes only one wavelength

To create **spatial coherence**: Pass through a narrow slit or pinhole to define the direction

**Laser light** typically has a very high degree of coherence
Two-Slit Interference

(a) Interference of light waves passing through two slits

When using a laser source, the monochromator and the first slit can be omitted, since the light is already coherent!
Stars are light emitting objects like our sun, but at a much greater distance, and thus appear point-like in the sky.

You let light from a bright star pass through a double slit, onto a sensitive detector. Do you expect to see interference fringes?

A. Yes, since the starlight has high spatial as well as spectral/temporal coherence.
B. No, since the starlight is spatially incoherent but spectrally/temporally coherent.
C. Since the starlight has high spatial but low spectral/temporal coherence, the interference pattern is visible only for a very short time.
D. Since the starlight has low spatial but high spectral/temporal coherence, interference will be seen only if the light first passes through a spatial filter (pinhole).
E. Since the starlight has high spatial but low spectral/temporal coherence, interference will be seen if the light first passes through a spectral/color filter.
Since the path difference is $dsin\theta$, and we get maxima when the difference is an integral number of wavelengths, when $m\lambda = dsin\theta$ or $sin\theta = m\lambda/d$.

The vertical positions of the bands on the screen become $y = R\tan\theta \approx R\sin\theta = mR\lambda/d$. 
An Actual 2-Slit Interference Pattern

Observe alternating light & dark bands.

maxima at $y \approx mR\lambda/d$.  
minima at $y \approx (m+\frac{1}{2})R\lambda/d$. 

Table:

<table>
<thead>
<tr>
<th>$m$ (constructive interference, bright regions)</th>
<th>$m + 1/2$ (destructive interference, dark regions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 $\rightarrow$</td>
<td>$\leftarrow 11/2$</td>
</tr>
<tr>
<td>4 $\rightarrow$</td>
<td>$\leftarrow 9/2$</td>
</tr>
<tr>
<td>3 $\rightarrow$</td>
<td>$\leftarrow 7/2$</td>
</tr>
<tr>
<td>2 $\rightarrow$</td>
<td>$\leftarrow 5/2$</td>
</tr>
<tr>
<td>1 $\rightarrow$</td>
<td>$\leftarrow 3/2$</td>
</tr>
<tr>
<td>0 $\rightarrow$</td>
<td>$\leftarrow 1/2$</td>
</tr>
<tr>
<td>$-1 \rightarrow$</td>
<td>$\leftarrow -1/2$</td>
</tr>
<tr>
<td>$-2 \rightarrow$</td>
<td>$\leftarrow -3/2$</td>
</tr>
<tr>
<td>$-3 \rightarrow$</td>
<td>$\leftarrow -5/2$</td>
</tr>
<tr>
<td>$-4 \rightarrow$</td>
<td>$\leftarrow -7/2$</td>
</tr>
<tr>
<td>$-5 \rightarrow$</td>
<td>$\leftarrow -9/2$</td>
</tr>
<tr>
<td></td>
<td>$\leftarrow -11/2$</td>
</tr>
</tbody>
</table>
Maxima and Minima

Assume that two sources are in phase.

For maxima, we want constructive interference:
\[ r_2 - r_1 = m\lambda, \text{ with } m = 0, \pm 1, \pm 2, \ldots \]

For minima, we want destructive interference:
\[ r_2 - r_1 = (m + \frac{1}{2})\lambda, \text{ with } m = 0, \pm 1, \pm 2, \ldots \]

There are alternatives - the two sources could be out of phase.
In this case we obtain minima where there were maxima, and vice versa.

More generally, there could be any phase difference between the two sources.
Radio Stations

Consider a radio or TV station with a single vertical dipole antenna. It will broadcast its signal equally in all directions along the surface.

But if we add a second antenna, the interference pattern results in more energy going out in some directions - so the signal can be received further away - and less going out in other directions.

In the configuration shown, interference reduces the energy going out in some sideways directions. If we changed the antennas to be out of phase, interference would reduce the energy going to the left and right.
You are going to use two antennas to broadcast radio mainly east/west from the center of Long Island. The antennas have to be east/west of each other. Should they be in phase or out of phase? How many wave lengths apart should they be? Choose the best answer

a) $0\lambda$ apart, in phase.
b) $0\lambda$ apart, out of phase.
c) $\frac{1}{2}\lambda$ apart, out of phase.
d) $\frac{1}{2}\lambda$ apart, in phase.
e) $1\lambda$ apart, phase does not matter.
Two point sources oscillate exactly out of phase. Point P is 7.3 wavelengths from the first source, and 4.8 wavelengths from the second source. As a result, at point P there is

a. Constructive interference.
b. Destructive interference.
c. Interference, but somewhere between constructive and destructive.
d. No interference at all
e. Not enough information to decide.