Physics 228 - Lecture 3

Today:
• Flat Mirrors
• Spherical Mirrors
• Lenses

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How does a mirror work?

Your image, in the mirror, appears to be the same height as you, and the same distance behind the mirror as you are in front.

Magnification: the ratio of the size of the image to the size of the object.

For a plane mirror, $$M = \frac{y'}{y} = 1.$$ 

This image is called a virtual image, as no light actually goes to or from the image position.
The woman in the drawing is looking at herself in a mirror. The top of the mirror is about even with her eyes, at a height $h$. How tall does the mirror have to be for her to see her feet?

a) $h$
b) $h/2$
c) $2h$
d) it depends on how far the mirror is from her

e) You cannot see your feet in a mirror
**Spherical Mirrors**

Plane mirrors are simpler, but non-plane mirrors are more fun.

In a spherical mirror, you can see a reflected image. The image can appear bigger or smaller than the object. The image can be upright or inverted. The image can be real or virtual.

How do we explain this? Geometry, and $\theta_{\text{reflected}} = \theta_{\text{incident}}$.

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M.C. Escher, self-portrait in mirrored sphere

“The point between your eyes is the absolute center. No matter how you turn or twist yourself, you can't get out of that central point.”

M.C. Escher
Concave Spherical Mirror

Parallel rays coming from infinity all reflect from the mirror and approximately meet at a point, the focal point, or focus.

(This is approximately true for a sphere as long as the radius $R$ is large compared to the separation of the rays from the optical axis.)

The closest distance between the focus and the mirror is called the focal length $f$. The focus is halfway between the center and the surface of the sphere:

$$f = \frac{R}{2}$$
Spherical Mirror: given an object, where is its image?

Is the image upright or inverted?

How do we use geometry to find the image?
**Finding the Image Using Principal Rays**

Put an object with base on optical axis.

Draw rays, reflect back from the mirror, see where they intersect.

Horizontal line from top reflects back through focal point.

Draw a line from the top through where the axis hits the mirror, and the reflection of that ray.

Draw a line from the top through the focus - it reflects back horizontally.

The image is where the rays intersect.
In this case, we see that we have an inverted, real image.

It is not necessary to draw all three rays to see where the focus is, but it is useful consistency check.
• Define the object distance $s$ and image distance $s'$. 

• Define the focal length $f$. 

• The image properties are different for $s > f$, $s = f$, and $s < f$. 

**Concave Spherical Mirror**

**Object** $s$ $s'$ $f$ 

**Image** $C$ $F$
Spherical Mirror - What happens as Object Moves?

From similar triangles, you can see that the magnification is $M = y'/y = -s'/s$. 

(a) Construction for $s = 30$ cm

All principal rays can be drawn. The image is inverted.

(b) Construction for $s = 20$ cm

Ray 3 (from Q through C) cannot be drawn because it does not strike the mirror. The image is inverted.

(c) Construction for $s = 10$ cm

Ray 2 (from Q through F) cannot be drawn because it does not strike the mirror.

(d) Construction for $s = 5$ cm

Parallel outgoing rays correspond to an infinite image distance. The image is virtual and erect.
Where is the Image?

Define:
Distance from mirror to focus = f = R/2.
Distance from mirror to object = s.
Distance from mirror to image = s'.

Then:
\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]
\[
s' = \frac{sf}{(s-f)}
\]

Spherical concave mirror:
s and f are > 0 (in front of the mirror), but where is s'?

For s > f: s' = \( \frac{sf}{(s-f)} \) > 0 \hspace{1cm} \text{(real image)}
For s = f: s' = \( \frac{sf}{(s-f)} = \infty \) \hspace{1cm} \text{(real image at infinity)}
For s < f: s' = \( \frac{sf}{(s-f)} < 0 \) \hspace{1cm} \text{(virtual image)}

For s' > 0, light comes together in front of mirror, a real image. For s' < 0, image is behind mirror, a virtual image (no light actually there).

We will use this equation to relate object and images in several contexts.
Return to the Flat Mirror

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \\
\frac{s'}{s} = \frac{sf}{(s-f)}
\]

Can we apply this to the flat mirror?

**Yes:** a flat mirror is like a concave mirror with \( R = \infty \).

Thus

\[
\frac{1}{s} + \frac{1}{s'} = 0 \\
\frac{s'}{s} = -1 \\
\frac{s'}{s} = -s
\]

The object is in front of the mirror, the (virtual) image is behind it.
The algebra for a convex mirror is the same as for a concave mirror, but $f < 0$ since the focus is on the other side of the mirror.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(with } f < 0)$$

Since $s$ is always positive and $f < 0$ for a convex mirror, the r.h.s. is $< 0$, so $s'$ is always negative.

Also, $|s'| < s$, so that the l.h.s. is negative as well. We have an upright virtual image behind the mirror, and the magnification $M = \frac{y'}{y} = -\frac{s'}{s} \leq 1$. 
What type of mirror produces a real, upright image?

a) plane  
b) convex  
c) concave, $s > f$  
d) concave, $s < f$  
e) none
¡Clicker: What is This?

a) Santa as he sees himself in a mirrored sphere.

b) Santa as he sees himself in a flat mirror after too much eggnog.

c) Santa’s reflection in a mirrored sphere, seen by an invisible elf sitting on his nose.

d) Prof. Zimmermann in 20 years.
We learned we can make a real image with mirrors, but that is not too helpful for putting a camera in your iphone.

For that, we will use lenses.

To "simplify" the geometry, we will assume lenses are made with spherical surfaces, and approximate that the size of the lens is small compared to its curvature.
Lenses

Thin glass lenses in air:

Many shapes are possible.

Generally, if the lens is thicker in the center, it causes parallel light rays to converge.

If the lens is thinner in the center, it causes light rays to diverge.

Let's see an example of what such a lens does... DEMO!
**Converging Lens**

Parallel rays converge to focal point.

Object forms real image on other side.
Diverging Lens

Parallel rays diverge as if coming from a point, the "focal point".

Object forms virtual image on same side
iClicker:

A converging lens projects a real image of an object onto a screen. Now I cover the lower half of the lens with a piece of cardboard. How will the image change?

a) The lower half of the image goes away.
b) The upper half of the image goes away.
c) There is no change.
d) The image becomes dimmer.
e) The image disappears.
Finding the position of the image - graphically

We use similar techniques to what we used with mirrors:

• Rays through the lens center (2) go straight.
• Rays parallel to axis (1) go through the focus on the image side.
• Rays through the focus on the object side (3) come out parallel to axis.
• The image is real and inverted.

(a) Converging lens
Finding the position of the image - graphically

For a diverging lenses, we again use similar techniques:

- Rays through the lens center go straight through. (2)
- Rays parallel to the axis (1) go (virtually) through the object focus.
- Rays pointing to the image focus (3) emerge parallel to symmetry axis.
- The image is virtual and upright.
I will move the object in front of a diverging lens upwards. How will the image change?

a) When the object is off axis the image goes away.
b) The image moves down.
c) The image focuses at a different distance, so it is not so sharp.
d) The image moves up.
e) The image does not change.
(b) Diverging lens
The curvature of the surfaces can be from the left or right, so we guess that the radii and the focal length $f$ can be positive or negative. How do we define the signs?

If the object is on the same side as the light going into the reflecting/refracting surface, $s > 0$.

If the image is on the same side as the light going out, $s' > 0$.

If the center of curvature is on the same side as the light going out, $R > 0$.
How do we determine the focal length?

"Lensmaker's equation":

\[
\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

\(R_1\) is the "incoming" side
\(R_2\) is the "outgoing" side

\(f > 0:\)

Converging lenses

Meniscus

Planoconvex

Double convex

\(R_{L,R} < 0\)

\(R_{R} < 0\)

\(R_{L} > 0\)

\(R_{R} < 0\)

\(f < 0:\)

Diverging lenses

Meniscus

Planoconcave

Double concave

\(R_{L,R} > 0\)

\(R_{R} > 0\)

\(R_{L} < 0\)

\(R_{R} > 0\)
Position of Image and Magnification

- Once we know the focal length $f$, and the object position $s$ we can find the image position $s'$ and the magnification $M$:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$M = \frac{y'}{y} = -\frac{s'}{s}$$

- The equations are the same ones we used for mirrors.

[PhET demo](https://phet.colorado.edu/en/sims/zoom-lens/) board examples
Finding the position of the image (converging lens)

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

As \( s \to \infty \), \( s' \to f \)
As \( s \to f \), \( s' \to \infty \)
If \( s = 2f \), \( s' = 2f \)

When \( s < f \), \( s' < 0 \) - we have a virtual upright image on the left side of the lens.
Finding the position of the image (diverging lens)

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

For a diverging lens, \( f < 0 \), so \( s' < 0 \): virtual image!
Microscopes and Telescopes

• Multiple optical elements:
• Objective lens + eyepiece
• Image formed by objective serves as object for eyepiece

Telescope:
Zoom Lens

(a) Zoom lens set for long focal length

(b) Zoom lens set for short focal length
I am going to turn a convex-flat lens around on the board, to make it flat-convex. What will happen?

(a) It still focuses, in a shorter length.
(b) It still focuses, at the same place.
(c) It still focuses, with a longer length.
(d) It will neither focus nor de-focus now. The rays stay parallel.
(e) It will act as a diverging lens now.