Physics 228

Today:
Nuclear Physics
Radioactivity
Optical Second Harmonic Generation

- Double the frequency of light inside a crystal.
- Two photons are converted to one photon at twice the frequency.
- Need crystal w/o inversion symmetry.
- “Phase matching”: Same refractive index at both frequencies, f and 2f.

Potassium Titanyl Phosphate (KTP)
Stability of Nuclei

Here we plot the lifetime of the nuclei (color, black is stable) in a chart of neutron number $N$ (y-axis) vs. proton number $Z$ (x-axis).

There is a narrow “valley of stability” around an optimal neutron-to-proton ratio.
Why do nuclei tend to have more neutrons than protons?

a) The neutrons have a magnetic attraction that the protons do not.
b) The neutrons form pairs but the protons do not.
c) The nuclear force is short range.

**d) Protons repel each other due to the Coulomb force.**

e) Protons are Fermions and go in different orbits, but neutrons are bosons and all can go into the lowest n = 1 ground state.
Nuclear Sizes

The volume of atomic nuclei is approximately proportional to the number of nucleons (think of a liquid droplet of nucleons).

Thus, the mass density of all nuclei is quite similar, $\rho \approx 2.3 \times 10^{17} \text{ kg/m}^3$ (compare with $10^3 \text{ kg/m}^3$ for water!)

As a consequence, the nuclear radii are a simple function of the number of nucleons $A$: $r \approx R_0 A^{1/3}$ with $R_0 = 1.2 \text{ fm.}$
Nuclear Masses

Nuclear masses are measured in a convenient unit: The atomic mass unit \( 1 \, \text{u} = 1.66053892 \times 10^{-27} \, \text{kg} \).

The a.m.u is defined as \( 1/12 \) of the mass of the \(^{12}\text{C}\) atom.

The mass of the proton is 1.0073 \( \text{u} \).

The mass of the neutron is 1.0087 \( \text{u} \).

The mass of the electron is 0.00055 \( \text{u} \).

The \(^{12}\text{C}\) atom consists of 6 electrons, 6 protons, and 6 neutrons, thus one would expect it to have a total mass of 12.099 \( \text{u} \). Yet it is only 12.000 \( \text{u} \). What is going on?

The nucleus consists of bound nucleons, not free nucleons. Binding in an attractive potential lowers the energy by an amount called the “binding energy”. But remember that mass is energy, and energy is mass, measured in different units (via \( E = mc^2 \)). Thus the mass of the nucleus will be lower than the mass of the free nucleons by this “binding energy”, converted to mass units.
The plot shows the total binding energy divided by the number of nucleons. The binding energy per nucleon exhibits maximum at $A = 62$.

Thus, if two light nuclei fuse into a heavier nucleus, energy is released (nuclear fusion in the sun).

If a heavy nucleus splits into two light ones, energy is released (fission, nuclear power plant).
By what factor is the radius of the nucleus $^{160}\text{Dy}$ larger than that of $^{20}\text{Ne}$?

a) $<1$ - because of the strength of the nuclear force, nuclei get smaller as you add more nucleons

b) 1 - all nuclei are about the same size

c) 2

d) 4

e) 8
Nuclear Excited States

It is possible to excite a single nucleon into a higher “orbit” (shell model), while leaving the state other nucleons unchanged. This is called a single-particle excitation.

Thinking of nuclei as liquid drops, you can also imagine the entire drop to vibrate and rotate (Just like molecules!). Since all nucleons contribute to these motions, these are called “collective” excitations.

In many nuclear spectra, there are in fact vibrational excited states of energy $(n+1/2)\hbar \omega$, and rotational excited states of energy $l(l+1)\hbar^2/2I$.

Some excited states are not as easy to classify. They are intermediate between single-particle and collective excitations, and are difficult to visualize. This is a manifestation of the difficult problem of “correlation” in quantum mechanics.
Natural Radioactivity

• γ decay: an excited state of a nucleus decays to a lower lying state, emitting a photon (γ-ray).

• α decay: an α-particle (consisting of 2 protons and 2 neutrons) is emitted.

• β decay: In a neutron-rich nucleus, a neutron is converted into a proton, emitting an electron (and another particle). A similar process may occur in proton-rich nuclei: A proton is converted into a neutron, emitting a positron (the positively charged anti-particle of the electron).

• electron capture: a proton is converted into a neutron by capturing an electron from an s-orbital. This is the only way for protons to convert to neutrons if there is insufficient energy to create a positron.
α Decay

Many heavy nuclei are energetically unstable for breakup into an α particle - a $^4$He nucleus - and a nucleus with $^{A-4}Z-2$. The (rest) mass of the parent nucleus is larger than the combined (rest) masses of the two products. What about the relativistic masses?

A useful way to think of this is as an α-particle bound into a potential well. The α moves around inside the potential, constantly reflecting off the wall. But its wave function has an exponential tail under the potential barrier, resulting in some probability of tunneling through the potential barrier.
β Decay

- Free protons are stable but free neutrons have a lifetime of about 880 s.
- The neutron’s decay is: $n \rightarrow p + e^- + \bar{\nu}_e$
- For historical reasons, in radioactive decay the electron is also referred to as a $\beta^-$ particle, and the positron as a $\beta^+$ particle.
- The energy given off in this reaction is 0.782 MeV. It corresponds to the (rest) mass difference between the neutron and the products.
- This process may also occur inside a neutron-rich nucleus (with a different lifetime and a different amount of energy released).
- The inverse process, $p \rightarrow n + e^+ + \nu_e$, is not energetically allowed for free protons, but it is allowed for protons in proton-rich nuclei.
Shown above is a “cut” through the Z vs. N nuclear energy landscape, for constant $A = Z + N$. The cut is perpendicular to the “valley of stability”.

Nuclei on the neutron-rich side of the valley approach the valley bottom by decaying in $\beta^-$ processes, while proton-rich nuclei decay toward the bottom via $\beta^+$ processes.

A related process is electron capture. Here an atomic electron enters the nucleus and the process $e + p \rightarrow n + \nu_e$ occurs in the nucleus, so $\beta^- + ^A Z \rightarrow ^A Z - 1 + \nu_e$.

In neutron stars, this process occurs to reduce the number of protons and increase the number of neutrons.
When nuclei decay they can decay to the ground state of the final nucleus, or to an excited state.

The excited states then decay by emitting photons (γ rays), to lower lying states or directly to the ground state.

Thus we end up in a different energy state of the same nucleus.
The rest mass of the proton is 1.0073 u.
The rest mass of the neutron is 1.0087 u.
The rest mass of the positron is 0.00055 u.
The rest mass of the neutrino is almost zero.

Why can a proton decay into a neutron, a positron, and a neutrino in some nuclei?

a) This is not possible as it would violate energy/mass conservation.
b) This is possible because the rest mass of the reactant (proton) is less than that of the products (neutron, positron, and neutrino).
c) This is possible because the relativistic mass is increased in the reaction.
d) This is possible in proton-rich nuclei because the reduced Coulomb repulsion in the final state makes the reaction exoenergetic (energy is given off).
e) This is possible because in neutron-rich nuclei, the Fermi energy of protons is higher than the Fermi energy of neutrons.