Physics 228

Today:

Stern Gerlach Experiment
Electron Spin
Pauli Exclusion Principle
Many Electron Atoms
Exam Grade Distribution

Report: "Phys220_SP19_Ex2"

<table>
<thead>
<tr>
<th>Quest#</th>
<th>Corr</th>
<th>NA</th>
<th>Wrong</th>
<th>%Corr</th>
<th>%Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>197</td>
<td>0</td>
<td>75</td>
<td>72</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>0</td>
<td>253</td>
<td>7</td>
<td>93</td>
</tr>
<tr>
<td>3</td>
<td>228</td>
<td>0</td>
<td>44</td>
<td>84</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>223</td>
<td>0</td>
<td>49</td>
<td>82</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>216</td>
<td>0</td>
<td>56</td>
<td>79</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>199</td>
<td>0</td>
<td>73</td>
<td>73</td>
<td>27</td>
</tr>
<tr>
<td>7</td>
<td>248</td>
<td>0</td>
<td>24</td>
<td>91</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>178</td>
<td>0</td>
<td>94</td>
<td>65</td>
<td>35</td>
</tr>
<tr>
<td>9</td>
<td>205</td>
<td>0</td>
<td>67</td>
<td>75</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>176</td>
<td>0</td>
<td>96</td>
<td>65</td>
<td>35</td>
</tr>
<tr>
<td>11</td>
<td>223</td>
<td>0</td>
<td>49</td>
<td>82</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
<td>138</td>
<td>3</td>
<td>131</td>
<td>51</td>
<td>48</td>
</tr>
<tr>
<td>13</td>
<td>116</td>
<td>2</td>
<td>154</td>
<td>43</td>
<td>57</td>
</tr>
<tr>
<td>14</td>
<td>166</td>
<td>1</td>
<td>105</td>
<td>61</td>
<td>39</td>
</tr>
<tr>
<td>15</td>
<td>124</td>
<td>1</td>
<td>147</td>
<td>46</td>
<td>54</td>
</tr>
<tr>
<td>16</td>
<td>223</td>
<td>0</td>
<td>49</td>
<td>82</td>
<td>18</td>
</tr>
</tbody>
</table>

**Quest#** - Question number from master copy
**Corr** - Number of students with correct answer to the question
**NA** - Number of students that did not answer the question
**Wrong** - Number of students with incorrect answer to the question
**%Corr** - Percent of students that answered correctly
**%Wrong** - Percent of students that answered incorrectly

Average = 66.2
Median = 68.8
Mavis is traveling at relativistic speed on a very long train. There are accurate clocks on each car. All the clocks on the train are synchronized with each other, in the train's frame. Stanley is standing on the platform as the train whizzes by, holding a clock in his hand. Every time one of the train's clocks passes by Stanley and his clock, he snaps a picture of both clocks next to each other. When comparing the clock readings,

A. Stanley and Mavis agree that Stanley's clock advances faster in comparison to the readings of Mavis' clocks, since Mavis' moving clocks are slowed by time dilation.

B. Stanley and Mavis agree that Stanley's clock advances more slowly in comparison to the readings of Mavis' clocks, since his clock is moving with respect to the train, and the measurements are taken in the train's reference frame (the synchronized clocks).

C. Stanley and Mavis agree that Stanley's clock advances at the same rate as the readings on Mavis' clocks.

D. It depends on who you ask: Stanley concludes that his clock runs fast, while Mavis concludes Stanley's clock to run slow, compared to the readings on Mavis' clocks.

E. This is an unsolvable paradox.
Space-Time Diagrams of Clocks in Space

Stanley's clocks

Stanley

Mavis

Stanley's clocks

Mavis

Mavis's clocks
As one of Mavis’s clocks zooms past Stanley’s synchronized clock array, we compare readings every time Mavis’s clock passes one of Stanley’s. We conclude Mavis’s clock runs slow.

Thus, from Stanley’s point of view, Mavis’s clock runs slow.

As one of Stanley’s clocks zooms past Mavis’s synchronized clock array, we compare readings every time Stanley’s clock passes one of Mavis’s. We conclude Stanley’s clock runs slow.

Thus, from Mavis’s point of view, Stanley’s clock runs slow.
The Stern-Gerlach Experiment

- If the magnetic field is not constant, there will be a force on the atom \( F = - \nabla U = - m_I \mu_B (dB/dz) \).
- An atom beam will separate into \( 2l+1 \) separate bunches if there is a \( B \) field with enough spatial variation.
- This shows the quantization of the \( z \)-component of the magnetic moment, and by inference, the quantization of \( L_z \).
The number of “bunches” should be $2l+1$, therefore odd.

This is observed for some atoms.

However, in other cases an even number is seen.

If we interpret this as angular momentum quantization, we require half-integer angular momentum quantum numbers $j = 1/2, 3/2, \text{etc.}$

The number of sublevels $2j+1$ is then even.
Electron Spin

- This half-integral angular momentum also shows up in the Zeeman effect (called “anomalous Zeeman effect”).
- Half-integral angular momentum is now understood as “spin” of the electron.
- Spin is an intrinsic angular momentum that the electrons always have, irrespective of orbital motion.
- For electrons, the magnitude of the spin angular momentum is
  \[ S = \sqrt{s(s + 1)}\hbar \quad \text{with} \quad s = \frac{1}{2} \] (spin quantum number).
- The z-component of the spin angular momentum is
  \[ S_z = m_s \hbar \quad \text{with} \quad m_s = \pm \frac{1}{2} \] (spin magnetic quantum number).
- For many particles (electrons, protons, neutrons, ...) \( s = \frac{1}{2} \). Such particles are called fermions.
- Other particles have integral spin (\( s = 0 \) for the Higgs boson, \( s = 1 \) for the photon). Such particles are called bosons.
Electron Spin Magnetic Moment

- For a classical spinning body of charge \(-e\) and mass \(m\) (assuming equal charge and mass distribution), we would expect a magnetic moment of \(\mu = (-e/2m)L\), where \(L\) would be replaced by the spin angular momentum \(S\).

- However, the electron is not a classical spinning body. Its magnetic moment is actually larger than expected classically:

\[
\mu = g(e/2m)S
\]

- The “quantum mechanics” factor \(g\) is called the “g-factor” (duh).

- Classical mechanics predicts \(g = -1\).

- Experimentally, \(g = -2.002319304361\) (the most precisely known fundamental constant in physics!!)

- Nonrelativistic QM (Schroedinger equation) makes no prediction at all. Spin has to be added in an ad-hoc manner (without derivation).

- Relativistic QM (Dirac equation) predicts \(g = -2\) (exactly).

- Another refinement of relativistic QM (quantum electrodynamics) predicts \(g = -2.002319304361\) (The most accurate theoretical prediction in physics!!)
As a result of spin, in each of the H-atom energy levels the electron can be "spin up" or "spin down". Thus, even for s-orbitals, the spin gives rise to a two-fold Zeeman splitting in a magnetic field.
What is the magnitude of the spin angular momentum of an electron?

a) 0
b) $\hbar$
c) $\hbar/2$
d) $+\hbar/2$ or $-\hbar/2$, depending on whether spin is “up” or “down”
e) $\sqrt{3\hbar}$

$$S = \sqrt{s(s + 1)}\hbar = \sqrt{\frac{1}{2} \left( \frac{1}{2} + 1 \right)} \hbar = \sqrt{\frac{3}{4}}\hbar$$
In heavier atoms, the nucleus has charge $Z e$.

The electrons in orbit about the nucleus repel each other.

This leads to "screening" of outer electrons by inner electrons: The potential is no longer $1/r$.

As a consequence, the energy levels of different $l$ quantum numbers (same $n$) are no longer degenerate.

Accurate calculations are much more complicated than for hydrogen ("Coulomb correlations").
Pauli Exclusion Principle

- The Pauli exclusion principle states that each single-particle state (characterized by the four quantum numbers \( n, l, m_l, m_s \)) can accommodate no more than one electron.

- As a consequence, as we add electrons to an atom, the inner shells “fill up” first, then the more weakly bound shells will be populated. (“Aufbau principle”).

- The Pauli exclusion principle applies to all half-integer spin particles, i.e., fermions (incl. protons and neutrons).

- There is no such principle for bosons. For example, a large number of photons may occupy the same quantum state in a laser.
Aufbau Principle

Periodic Table of the Elements