Recitations start this week, with quiz. Also: Labs 230!

1st homework due
Refraction

Snell's Law

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ \frac{\sin \theta_1}{\sin \theta_2} = n \]

The medium with larger index has the smaller angle - the ray is closer to the normal.
Derivation of Snell's Law

\[
\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{d \sin(\theta_2)}{d \sin(\theta_1)}
\]

\[
n_1 \sin(\theta_1) = n_2 \sin(\theta_2)
\]

\[
\lambda_1 = \frac{\lambda_0}{n_1}
\]

\[
\lambda_2 = \frac{\lambda_0}{n_2}
\]
Law of reflection

Apply Snell’s law to the incident and reflected waves:

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]

In reflection \( n_1 = n_2 \), thus \( \theta_1 = \theta_2 \).
Due to refraction, objects get distorted, or are not where they appear to be...
Due to refraction, objects get distorted, or are not where they appear to be...
Light passes from air (n = 1) into glass (n = 1.5). Which of the following is true?

a) $\lambda_{\text{air}} > \lambda_{\text{glass}}$, $f_{\text{air}} < f_{\text{glass}}$

b) $\lambda_{\text{air}} > \lambda_{\text{glass}}$, $f_{\text{air}} > f_{\text{glass}}$

c) $\lambda_{\text{air}} > \lambda_{\text{glass}}$, $f_{\text{air}} = f_{\text{glass}}$

d) $\lambda_{\text{air}} < \lambda_{\text{glass}}$, $f_{\text{air}} > f_{\text{glass}}$

e) $\lambda_{\text{air}} < \lambda_{\text{glass}}$, $f_{\text{air}} < f_{\text{glass}}$
Snell's Law: \( \sin(\theta_{\text{out}}) = n \sin(\theta_{\text{in}}) \)

- The largest \( \theta_{\text{out}} \) can get is 90°.
- This corresponds to \( \sin(\theta_{\text{out}}) = 1 \).
- Thus \( \sin(\theta_{\text{in,\text{max}}}) = \frac{1}{n} \) according to Snell's law.
- The largest \( \theta_{\text{in}} \) for which Snell’s law can have any meaning is \( \theta_{\text{in,\text{max}}} = \arcsin\left(\frac{1}{n}\right) \).
- For \( n = 1.5 \), we find \( \theta_{\text{in,\text{max}}} = 42° \).

So what if the incident ray comes in at \( \theta_{\text{in}} > 42° \)? This would imply \( \sin(\theta_{\text{out}}) > 1 \), which is impossible. What happens?
A: All the light is reflected!

“Total Internal Reflection”
Shown in TIR demos
Phet simulation.
Total Internal Reflection

(a) Total internal reflection

Total internal reflection occurs only if $n_b < n_a$.

At the critical angle of incidence, $\theta_{crit}$, the angle of refraction $\theta_b = 90^\circ$.

Any ray with $\theta_a > \theta_{crit}$ shows total internal reflection.

Ordinary reflection: Some light is transmitted and some reflected.

(b) Total internal reflection demonstrated with a laser, mirrors, and water in a fishbowl

Incident laser beams

Reflected at interface

Total internal reflection

Mirror
Total Internal Reflection

(a) Total internal reflection occurs only if $n_b < n_a$.

(b) Total internal reflection demonstrated with a laser, mirrors, and water in a fishbowl.

- **TIR**: There is no transmitted beam.
- **Critical angle for TIR**: $\sin(\theta_{crit}) = \frac{n_2}{n_1}$ (with $n_1 > n_2$)
- **TIR** can occur only when light is directed from the “inside” of an optically denser medium (greater $n$) to the outside, thus “internal” reflection.
Applications of Total Internal Reflection

(a) Total internal reflection in a Porro prism

If the incident beam is oriented as shown, total internal reflection occur on the 45° faces (because, for a glass–air interface, θ_{crit} = 41.1°).

- Optical fibers
- Retroreflectors
- Pretty jewelry
- Binoculars
- Fingerprint readers
Dispersion

- Refractive index changes with wavelength
- Different wavelengths refract by different angles
- Separation of colors when light passes a prism
Rainbows

Sun + Rain = Rainbow!
Rainbows

Incident sunlight (white)

Refraction

Colors coming out

Total Internal Reflection
The woman is wondering why the mirror reverses her left/right, but not up/down. (This confuses nearly everyone!) What would you tell her?

a) I don’t know why.
b) It’s because your eyes are left/right, not up/down.
c) It does reverse you up/down if you lie down sideways.
d) It doesn’t reverse you left/right or up/down, it reverses you front/back.
e) It does reverse you up/down too.
Reflections from a Mirror

\[ \theta_{\text{reflected}} = \theta_{\text{incident}} \]

Looking to the right, she sees further to the right.

Looking to the left, she sees further to the left.

Looking up, she sees further up.

Looking down, she sees further down.
Reflections from a Mirror

$\theta_{\text{reflected}} = \theta_{\text{incident}}$

The more in front of the mirror the object is, the more behind you look to see its reflection.

The mirror reverses front - back.
The woman in the drawing is looking at herself in a mirror. The top of the mirror is about even with her eyes, at a height $h$. How tall does the mirror have to be for her to see her feet?

a) $h$

b) $h/2$

c) $2h$

d) it depends on how far the mirror is from her

e) You cannot see your feet in a mirror
How does a mirror work?

Your image, in the mirror, appears to be the same height as you, and the same distance behind the mirror as you are in front.

Magnification: the ratio of the size of the image to the size of the object.

For a plane mirror, \( M = y'/y = 1. \)

This image is called a virtual image, as no light actually goes to or from the image position.
Spherical Mirrors

Plane mirrors are simpler, but non-plane mirrors are more fun.

In a spherical mirror, you can see a reflected image. The image can appear bigger or smaller than the object. The image can be upright or inverted. The image can be real or virtual.

How do we explain this? Geometry, and $\theta_{\text{reflected}} = \theta_{\text{incident}}$.

M.C. Escher, self-portrait in mirrored sphere

“The point between your eyes is the absolute center. No matter how you turn or twist yourself, you can't get out of that central point.”

M.C. Escher
Concave Spherical Mirror

Parallel rays coming from infinity all reflect from the mirror and approximately meet at a point, the focal point, or focus.

(This is approximately true for a sphere as long as the radius $R$ is large compared to the separation of the rays from the optical axis.)

The closest distance between the focus and the mirror is called the focal length $f$. The focus is halfway between the center and the surface of the sphere:

\[ f = \frac{R}{2} \]
Spherical Mirror: given an object, where is its image?

Is the image upright or inverted?

How do we use geometry to find the image?
Put an object with base on optical axis.

Draw rays, reflect back from the mirror, see where they intersect.

Horizontal line from top reflects back through focal point.

Draw a line from the top to where the axis hits the mirror, and the reflection of that ray.

Draw a line from the top through the focus - it reflects back horizontally.

Draw a line from the top of object through the center of the spherical mirror, along radius, it reflects back on its incoming path.

The image is where the rays intersect.
Finding the Image Using Principal Rays

In this case, we see that we have an inverted, real image.

It is not necessary to draw all four rays to see where the focus is, but it is useful to draw at least three as a consistency check.
• Define the positions of the object and image as $s$ and $s'$. 

• Define the focal length $f$. 

• The image properties are different for $s > f$, $s = f$, and $s < f$. 

For $s > f$, as you move the object in towards the focus, the image moves out and gets bigger. It is a "real" inverted image - it would appear there on a piece of paper.
At the focus, the rays are parallel: The image is at $\infty$. 

Spherical Mirror - What Happens as Object Moves?

(c) Construction for $s = 10$ cm

Ray 2 (from $Q$ through $F$) cannot be drawn because it does not strike the mirror.

Parallel outgoing rays correspond to an infinite image distance.
Inside the focus, the image is virtual (no light at the image) & upright.
a) Santa as he sees himself in a mirrored sphere.

b) Santa as he sees himself in a flat mirror after too much eggnog.

c) Santa’s reflection in a mirrored sphere, seen by an invisible elf sitting on his nose.

d) Prof. Zimmermann in 20 years.

Because the point between the observer’s eyes should be in the center, Santa is not the observer!