Physics 228

Today:
Uncertainty Principle
Blackbody Radiation
Matter Waves
Heisenberg's Uncertainty Principle

More formal theory (which precisely defines “typical”), results in Heisenberg's uncertainty principle:

\[
\begin{align*}
\Delta x \Delta p_x & \geq \hbar/2 \\
\Delta y \Delta p_y & \geq \hbar/2 \\
\Delta z \Delta p_z & \geq \hbar/2
\end{align*}
\]

Here, \( \Delta \) means “uncertainty” in position or momentum, and \( \hbar = h/2\pi = 1.055 \times 10^{-34} \) J·s.

Uncertainties in different directions are unrelated. For example, \( \Delta x \Delta y \) could be 0, \( \Delta y \Delta p_z \) could be 0, and \( \Delta p_y \Delta p_z \) could also vanish.
Heisenberg's Uncertainty Principle

- As in our toy derivation, the uncertainty results from the wave nature of light.
- There is no way in principle to avoid it. This was considered in a series of thought experiments in the 1920s and 1930s, and also comes out of the formal mathematics of quantum mechanics.
- We will see later that the same uncertainty principle also applies to matter (electrons, nuclei, atoms).
Momentum / Position Localization

- Consider the light wave heading towards the slit, with electric field \( E = E(x,t) = E_0 \sin(kx-\omega t) \).
- This wave has energy \( E = hf = \hbar \omega \), and momentum \( p = \hbar k \).
- This is a wave of definite momentum \( p \), but we have no idea where “the photon” is located - it extends out uniformly over all \( x \).
- This is a "feature" of the sine wave - you pick a definite \( k \) or \( p \), and the wave extends uniformly over all \( x \).
Wave Packets

Instead of a wave extending to positive and negative infinity, consider a wave packet of spatial extent $\Delta x$:

• The Fourier transform of such a wave packet extends in $k$-space by an amount $\Delta k$, with $\Delta x \Delta k \geq \frac{1}{2}$.
• The uncertainties $\Delta$ refer to the standard deviation of the probability distribution (squared wave function).
• This is a mathematical statement (about Fourier transforms), not a physical one.
• By converting wavevector $k$ to momentum via $p = \hbar k$, we get the physical uncertainty principle: $\Delta x \Delta p \geq \hbar/2$. 
Uncertainty Principle: Energy and Time

We have just seen that \( p \) and \( x \) are "conjugate" variables. But our equation for the traveling wave

\[
E = E_0 \sin \left( \frac{px}{\hbar} - \frac{Et}{\hbar} \right)
\]

has two pairs of arguments, \((p \ x)\) and \((E \ t)\). What about localization in energy and time?

Answer: Energy - time uncertainty applies:

\[
\Delta E \ \Delta t \geq \frac{\hbar}{2}
\]

This is relevant for excited states (or particles) with a finite lifetime \( \Delta t \). The excited state energy (or the particle’s mass, via \( E = mc^2 \)) is then subject to the above uncertainty.
In a two-slit interference experiment, a single photon is detected in the upper half of the screen. The photon must have passed through

a. the upper slit  
b. the lower slit  
c. neither slit  
d. both slits  
e. impossible to decide which slit
Blackbody Radiation: What is a Blackbody?

• Blackbody radiation is the thermal radiation (light) emitted by a hot “blackbody”.

• A blackbody is an object that does not reflect or transmit any incident light. All incident light is absorbed: a cold blackbody is perfectly black. (If it is very hot it will be red, orange, yellow, or white.)

• No real materials behave in this way. A small amount of light is always reflected, even for the blackest soot or paint.

• However, we can make an artificial “black body” by considering a sufficiently large cavity in any opaque material, with a small opening.

• All light entering the opening will bounce around randomly inside the cavity, losing some of the photons at each bounce, and no light escapes back out.

• Demo: a “blackbody box”.
Blackbody Radiation

- The thermal radiation inside any cavity or box can be described as a set of standing waves or “normal modes”.

<table>
<thead>
<tr>
<th>Radiation modes in a hot cavity provide a test of quantum theory</th>
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<tbody>
<tr>
<td>#Modes per unit frequency per unit volume</td>
</tr>
<tr>
<td><strong>CLASSICAL</strong></td>
</tr>
<tr>
<td><strong>QUANTUM</strong></td>
</tr>
</tbody>
</table>

- Classical physics predicts that in thermal equilibrium, every normal mode should contain, on average, an amount of energy equal to $k_B T$ (“equipartition theorem”).

- From this we can predict the spectral energy density of the light emitted from the blackbody (cavity). The problem is, the prediction does not agree with experiment! (Total energy predicted to be infinite - oops! “Ultraviolet catastrophe”)

- If we assume, on the other hand, that each normal mode is populated by “lumps” of energy $E = hf$ (i.e., photons), where the number of such photons is determined by the rules of statistical mechanics, the correct spectral energy distribution results. Sweet.

- The formula for the spectral energy distribution emitted by a blackbody is named “Planck law”, after the guy who figured this out.
Blackbody Radiation - Planck Law

![Graph of Blackbody Radiation]

- **UV**
- **VISIBLE**
- **INFRARED**

Spectral radiance (kW · sr⁻¹ · m⁻² · nm⁻¹)

- **5000 K**
- **4000 K**
- **3000 K**

Classical theory (5000 K)

Wavelength (μm)
What is the significance of “black” in “blackbody”? Why don’t we have a celebrated law for “whitebody radiation”, “redbody radiation”, “greenbody radiation”, or whatever your favorite color is? And what about the Fifty Shades of Grey?

a) Physicists are boring. They cannot appreciate the beauty of colors. Or shades of grey.

b) Only black bodies radiate thermal radiation. Other colors won’t.

c) It is illegal to discriminate between black bodies and white bodies. Thus all bodies are subject to the same law.

d) Each color has its own radiation law. The non-black laws require understanding of the theory of QCD (quantum chromodynamics, chromos = color).

e) The radiation inside any closed cavity, no matter what the color of the material, is identical to that emitted from a perfect black body.
Quantum Weirdness

• Wave-Particle Duality: Waves are also particles. Particles are also waves.

• Uncertainty Principle: Precise knowledge of position and momentum of a photon, at the same time, is not possible. This applies to all particles, not just photons.

• Rather than telling us what will definitely happen in a given experiment, quantum mechanics only tells us the probability of certain outcomes.

• In the two-slit experiment, we cannot say which slit a photon goes through. If there is interference, it goes through both.

• We will see that this is true for electrons and other particles as well.
Wave Particle Duality

In 1924, Louis de Broglie proposed that, since light acts as both a particle and a wave, classical particles such as electrons also act as both particles and waves. “Matter Waves”

For matter waves, we have the same relations between momentum $p$ and wavelength $\lambda$ as for light:

$$p = \frac{h}{\lambda}$$

The uncertainty relations we discussed for photons - $\Delta E \Delta t \geq \frac{\hbar}{2}$, $\Delta x \Delta p_x \geq \frac{\hbar}{2}$, etc. also apply to particles.

But we have different relations between energy $E$ and momentum $p$ since the photon is massless:

- photons: $E = pc$
- Massive particles (relativistic): $E^2 = (pc)^2 + (mc^2)^2$
- Massive particles (non-relativistic): $K = \frac{p^2}{2m}$
## Wave Particle Duality Summary

<table>
<thead>
<tr>
<th>Photons</th>
<th>Electrons, etc.</th>
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<tbody>
<tr>
<td>$E = hf$</td>
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</tr>
<tr>
<td>$p = \frac{h}{\lambda} = \hbar k$</td>
<td>$p = mv = \frac{h}{\lambda} = \hbar k$</td>
</tr>
<tr>
<td>$p = \frac{E}{c}$</td>
<td>$p = (2mK)^{1/2}$</td>
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</table>
Experimental Confirmation

The wave nature of electrons was demonstrated within a few years of its prediction by Davisson and Germer, by measuring diffraction of electrons by the atoms on the surface of a Ni crystal:
What is the wavelength of an electron, moving at $v = 0.9 \, c$.

$p = \gamma mv = 5.64 \times 10^{-23} \, \text{kg} \cdot \text{m/s}$.

$\lambda = \frac{h}{p} = 6.626 \times 10^{-34} \, \text{Js} / 5.64 \times 10^{-23} \, \text{kg} \cdot \text{m/s} \approx 10^{-11} \, \text{m}$.

Remember the diffraction limit for imaging:

$$\sin \theta = 1.22 \frac{\lambda}{D}.$$  

Since the focal length of a lens cannot be made much smaller than the diameter, the smallest object that can be resolved in a microscope is about the size of the wavelength used.

Electron wavelengths are similar to atomic sizes, so electrons are useful for imaging atoms: Electron microscope.

The size of an atom is $\approx 10^{-10} \, \text{m}$.

The size of an atomic nucleus is $\approx 10^{-15} \, \text{m}$.

The size of an electron is zero (point particle, for all we know).
Electron Microscope

- Similar to optical microscope in principle, just replace light by electrons.
- For lenses, use appropriately shaped electric fields.
- Useful for imaging atoms, cells, viruses, nanostructures, …
iClicker

What is your wavelength, as you are running along at 10 m/s?

\[ p = mv = 70 \text{ kg} \times 10 \text{ m/s} = 700 \text{ kg} \cdot \text{m/s}. \]

\[ \lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ Js}}{700 \text{ kg} \cdot \text{m/s}} \approx 10^{-36} \text{ m}. \]

Can we observe interference/diffraction?

a. Sure. Just give me a fine enough diffraction grating.

b. Two slit interference would require you to run through two doors at the same time. This is only possible in science-fiction movies.

c. This works only for people with split-personality disorder.

d. Wave-particle duality applies only to microscopic particles, not to macroscopic bodies.

e. Impossible to observe in practice. Wavelength is many orders of magnitude smaller than size of human body.