Physics 228

Today:
Uncertainty Principle
Blackbody Radiation
Matter Waves
Heisenberg's Uncertainty Principle

The uncertainty principle states that the uncertainty of a particle's position $Δx$ and the corresponding momentum uncertainty $Δp_x$ cannot be arbitrarily small:

$$Δx \ Δp_x ≥ \hbar/2$$

Here, $Δ$ means “uncertainty” in position or momentum, and $\hbar = h/2π = 1.055x10^{-34}$ J·s.

This is, of course, unlike anything encountered in classical mechanics, and quite counter-intuitive for point particles. It makes a lot more sense for waves, though!

We will illustrate how this comes about in a particular situation (photons passing through a slit), but it is true for all particles in all situations!
Consider a photon going through a single slit of width a. Almost all of the light goes into a broad central maximum of width $\theta = \lambda/a$.

Since we see photons diffracted at some non-zero angle, they have a (small) sideways component of their momentum, with a “typical” magnitude $p_y = \mu \theta p = \mu (\lambda/a) p$. ($\mu$ is a number of order unity, which is determined by what exactly we mean by “typical”.)

The momentum of photons is related to the wavelength by $p = E/c = hf/c = h/\lambda$. We now plug this in for $p$ above.

We find $p_y = \mu (\lambda/a)(h/\lambda) = \mu h/a$, or $p_y a = \mu h$.

The “uncertainty” of the transverse momentum (resulting from going through the slit) multiplied by the “uncertainty” of the transverse position (the width of the slit) is of the order of Planck's constant $h$. 
Uncertainty Principle

- If we try to localize the photon position by making a smaller slit, the phenomenon of diffraction leads to a wider central maximum, $\theta \approx \lambda/a$, and a consequent greater variation in sideways momentum, since $p_y \approx h/a$.

- This idea is inconsistent with the classical idea that the position and the momentum of a particle can in principle both be determined exactly.

- We see that the better we determine one of these quantities, by for example making the slit smaller, the worse we determine the other.
Heisenberg's Uncertainty Principle

More formal theory (which precisely defines “typical”), results in Heisenberg's uncertainty principle:

\[
\Delta x \Delta p_x \geq \hbar/2 \\
\Delta y \Delta p_y \geq \hbar/2 \\
\Delta z \Delta p_z \geq \hbar/2 
\]

Here, \(\Delta\) means “uncertainty” in position or momentum, and \(\hbar = h/2\pi = 1.055\times10^{-34}\ \text{J}\cdot\text{s}\).

Uncertainties in different directions are unrelated. For example, \(\Delta x\Delta y\) could be 0, \(\Delta y\Delta p_z\) could be 0, and \(\Delta p_y\Delta p_z\) could also vanish.
Heisenberg's Uncertainty Principle

- As in our toy derivation, the uncertainty results from the wave nature of light.
- There is no way in principle to avoid it. This was considered in a series of thought experiments in the 1920s and 1930s, and also comes out of the formal mathematics of quantum mechanics.
- We will see later that the same uncertainty principle also applies to matter (electrons, nuclei, atoms).
• Consider the light wave heading towards the slit, with electric field \( E = E(x,t) = E_0 \sin(kx-\omega t). \)

• This wave has energy \( E = hf = \hbar \omega, \) and momentum \( p = \hbar k. \)

• This is a wave of definite momentum \( p, \) but we have no idea where “the photon” is located - it extends out uniformly over all \( x. \)

• This is a "feature" of the sine wave - you pick a definite \( k \) or \( p, \) and the wave extends uniformly over all \( x. \)
Wave Packets

Instead of a wave extending to positive and negative infinity, consider a wave packet of spatial extent $\Delta x$:

- The Fourier transform of such a wave packet extends in $k$-space by an amount $\Delta k$, with $\Delta x \Delta k \geq \frac{1}{2}$.
- The uncertainties $\Delta$ refer to the standard deviation of the probability distribution (squared wave function).
- This is a mathematical statement (about Fourier transforms), not a physical one.
- By converting wavevector $k$ to momentum via $p = \hbar k$, we get the physical uncertainty principle: $\Delta x \Delta p \geq \hbar/2$. 
Uncertainty Principle: Energy and Time

We have just seen that $p$ and $x$ are "conjugate" variables. But our equation for the traveling wave

$$E = E_0 \sin\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right)$$

has two pairs of arguments, $(p, x)$ and $(E, t)$. What about localization in energy and time?

Answer: Energy - time uncertainty applies:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

This is relevant for excited states (or particles) with a finite lifetime $\Delta t$. The excited state energy (or the particle's mass, via $E = mc^2$) is then subject to the above uncertainty.
Blackbody Radiation: What is a Blackbody?

• Blackbody radiation is the thermal radiation (light) emitted by a hot “blackbody”.

• A blackbody is an object that does not reflect or transmit any incident light. All incident light is absorbed: a cold blackbody is perfectly black. (If it is very hot it will be red, orange, yellow, or white.)

• No real materials behave in this way. A small amount of light is always reflected, even for the blackest soot or paint.

• However, we can make an artificial “black body” by considering a sufficiently large cavity in any opaque material, with a small opening.

• All light entering the opening will bounce around randomly inside the cavity, losing some of the photons at each bounce, and no light escapes back out.

• Demo: a "blackbody box".
Blackbody Radiation

- The thermal radiation inside any cavity or box can be described as a set of standing waves or “normal modes”.

- Classical physics predicts that in thermal equilibrium, every normal mode should contain, on average, an amount of energy equal to \( k_B T \) (“equipartition theorem”).

- From this we can predict the spectral energy density of the light emitted from the blackbody (cavity). The problem is, the prediction does not agree with experiment! (Total energy predicted to be infinite - oops! “Ultraviolet catastrophe”)

- If we assume, on the other hand, that each normal mode is populated by “lumps” of energy \( E = hf \) (i.e., photons), where the number of such photons is determined by the rules of statistical mechanics, the correct spectral energy distribution results. Sweet.

- The formula for the spectral energy distribution emitted by a blackbody is named “Planck law”, after the guy who figured this out.
Blackbody Radiation - Planck Law

The diagram illustrates the spectral radiance of blackbody radiation at different temperatures. The y-axis represents the spectral radiance in kW·sr⁻¹·m⁻²·nm⁻¹, and the x-axis represents the wavelength in micrometers (µm). The different curves correspond to temperatures of 3000 K, 4000 K, and 5000 K, with the latter being the classical theory curve. The visual representation shows a peak in the visible spectrum for the 5000 K case, indicating the maximum emission at this temperature.
What is the significance of “black” in “blackbody”? Why don’t we have a celebrated law for “whitebody radiation”, “redbody radiation”, “greenbody radiation”, or whatever your favorite color is? And what about the Fifty Shades of Grey?

a) Physicists are boring. They cannot appreciate the beauty of colors. Or shades of grey.

b) Only black bodies radiate thermal radiation. Other colors won’t.

c) We don’t discriminate between black bodies and white bodies. All bodies are subject to the same law!

d) Each color has its own radiation law. The non-black laws require understanding of the theory of QCD (quantum chromodynamics, chromos = color).

e) The radiation inside any closed cavity, no matter what the color of the material, is identical to that emitted from a perfect black body.
In a two-slit interference experiment, a single photon is detected in the upper half of the screen. The photon must have passed through

a. the upper slit
b. the lower slit
c. neither slit
d. both slits
e. impossible to decide which slit
Quantum Weirdness

• Waves are also particles. Particles are also waves. (True for all particles, not just photons.)

• Uncertainty Principle: Precise knowledge of position and momentum of a particle, at the same time, is not possible.

• In the two-slit experiment, we cannot say which slit a photon goes through. If there is interference, the “which-slit” question is indeterminate.

• Rather than telling us what is “really” going on at the microscopic level (like: where is the particle when we are not looking at it), quantum mechanics just gives us information about the outcome of future experiments.

• Rather than telling us what will definitely happen in a given experiment, quantum mechanics only tells us the probability of certain outcomes.
Matter Waves

In 1924, Louis de Broglie proposed that, since light acts as both a particle and a wave, classical particles such as electrons also act as both particles and waves. “Matter Waves”

For matter waves, we have the same relations between momentum $p$ and wavelength $\lambda$ as for light:

$$p = \frac{h}{\lambda}$$

The uncertainty relations we discussed for photons - $\Delta E \Delta t \geq \frac{\hbar}{2}$, $\Delta x \Delta p_x \geq \frac{\hbar}{2}$, etc. also apply to particles.

But we have different relations between energy $E$ and momentum $p$ since the photon is massless:

- photons: $E = pc$
- Massive particles (relativistic): $E^2 = (pc)^2 + (mc^2)^2$
- Massive particles (non-relativistic): $K = \frac{p^2}{2m}$
# Massless vs. Massive Particles

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<thead>
<tr>
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<th>Photons</th>
<th>Electrons, etc. (nonrelativistic)</th>
<th>Electrons, etc. (relativistic)</th>
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</thead>
<tbody>
<tr>
<td><strong>Energy (E)</strong></td>
<td>$E = hf$</td>
<td>$E = hf$</td>
<td>$E = hf$</td>
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<tr>
<td><strong>Momentum (p)</strong></td>
<td>$p = h/\lambda = \hbar k$</td>
<td>$p = mv = h/\lambda = \hbar k$</td>
<td>$p = \gamma mv = h/\lambda = \hbar k$</td>
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<tr>
<td><strong>Relativistic</strong></td>
<td>$p = E/c$</td>
<td>$p^2 = 2mK$</td>
<td>$p^2 = (E/c)^2 - m^2c^2$</td>
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Experimental Confirmation

The wave nature of electrons was demonstrated within a few years of its prediction by Davisson and Germer, by measuring diffraction of electrons by the atoms on the surface of a Ni crystal:
What is the wavelength of an electron, moving at $v = 0.9 \, c$.

$$p = \gamma m v = 5.64 \times 10^{-23} \text{ kg} \cdot \text{m/s}.$$  

$$\lambda = \frac{h}{p} = 6.626 \times 10^{-34} \text{ Js} / 5.64 \times 10^{-23} \text{ kg} \cdot \text{m/s} \approx 10^{-11} \text{ m}.$$  

Remember the diffraction limit for imaging:

$$\sin \theta = 1.22 \frac{\lambda}{D}.$$  

Since the focal length of a lens cannot be made much smaller than the diameter, the smallest object that can be resolved in a microscope is about the size of the wavelength used.

Electron wavelengths are similar to atomic sizes, so electrons are useful for imaging atoms: Electron microscope.

The size of an atom is $\approx 10^{-10} \text{ m}.$

The size of an atomic nucleus is $\approx 10^{-15} \text{ m}.$

The size of an electron is zero (point particle, for all we know).
Electron Microscope

- Similar to optical microscope in principle, just replace light by electrons.
- For lenses, use appropriately shaped electric fields.
- Useful for imaging atoms, cells, viruses, nanostructures, ...

[Diagram of electron microscope components showing a high-voltage supply, vacuum chamber, cathode, accelerating anode, condensing lens, objective lens, intermediate image, projection lens, and final image.]