Physics 228 Today:

Lorentz Transform (LT) of Velocity
LT and Simultaneity
LT and Time Dilation
LT and Length Contraction
Doppler Effect
Review: Lorentz Transformation

\[
\begin{align*}
    x' &= \gamma (x - u t) \\
    y' &= y \\
    z' &= z \\
    t' &= \gamma (t - u x / c^2)
\end{align*}
\]

\[
\begin{align*}
    x &= \gamma (x' + u t') \\
    y' &= y' \\
    z &= z' \\
    t &= \gamma (t' + u x'/c^2)
\end{align*}
\]

Mavis moving at \( u = 0.6 \, c \)

(lines of simultaneity are horizontal in Stanley’s frame)
When constructing this space-time diagram, how would we find Mavis's world line?

a) By setting $x = 0$ in the LT.
b) By setting $t = 0$ in the LT.
c) By setting $x' = 0$ in the LT.
d) By setting $t' = 0$ in the LT.
e) None of the above.
In nonrelativistic physics, velocities just add. In relativity, this can't be the case, otherwise a light beam that was emitted by a moving source would move faster than c. How do we transform velocities?

Let's say we have a bird flying at velocity \( v \) in Stanley's frame, corresponding to velocity \( v' \) in Mavis's frame. (As always, Mavis is moving with velocity \( u \) with respect to Stanley.)

The transverse \((y, z)\) components of velocity are, of course, the same in the two frames. To transform the \( x \)-component, let's pick 2 events on the world line of the bird, and transform them into Mavis' frame:
Lorentz Transform of Velocity

In Stanley’s frame the space interval between events A and B is \( \Delta x \), and the time interval is \( \Delta t \).

In Mavis’s frame, those intervals are \( \Delta x' \) and \( \Delta t' \), respectively:

\[
\begin{align*}
  x_A' &= \gamma (x_A - u t_A) \\
  x_B' &= \gamma (x_B - u t_B)
\end{align*}
\]

\[
\begin{align*}
  (x_A' - x_B') &= \gamma [(x_A - x_B) - u (t_A - t_B)] \\
  \Delta x' &= \gamma (\Delta x - u \Delta t) \\
  \Delta t' &= \gamma (\Delta t - u \Delta x/c^2)
\end{align*}
\]

\[
v' = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x - u \Delta t}{\Delta t - u \Delta x/c^2}
\]

Now divide through by \( \Delta t \), and recall that \( \Delta x / \Delta t = v \):

\[
\begin{align*}
v' &= \frac{v - u}{1 - uv/c^2} \quad \text{(Lorentz velocity transformation)} \\
v &= \frac{v' + u}{1 + uv'/c^2}
\end{align*}
\]
Lorentz Transform and Simultaneity

Consider two events that are simultaneous in Stanley’s frame: Event 1 at \( x = y = z = t = 0 \), and event 2 at \( x = L, y = z = t = 0 \). Are they simultaneous in Mavis’ frame?

Apply Lorentz Transformation:

\[
x' = \gamma (x - ut) \quad y' = y \quad z' = z \quad t' = \gamma (t - ux/c^2)
\]

We obtain for event 1 that \( x' = y' = z' = t' = 0 \).

We obtain for event 2 that \( y' = z' = 0 \), but \( x' = \gamma L \), and \( t' = -\gamma u L/c^2 \).

In Stanley’s frame, the events are simultaneous but a distance \( L \) apart. In Mavis’ frame, the events are separated in space as well as in time!
Lorentz Transform and Time Dilation

Is time dilation consistent with the Lorentz transform?

Consider two events at the same position in Mavis’ frame, at different times (e.g., Mavis’ heart beating twice):
event 1 at \( x' = y' = z' = t' = 0 \), and event 2 at \( x' = y' = z' = 0, \ t' = T \).

What are the coordinates in Stanley’s frame?

Apply Lorentz Transformation:

\[
x = \gamma(x' + ut') \quad y = y' \quad z = z' \quad t = \gamma(t' + ux'/c^2)
\]

We obtain for event 1 that \( x = y = z = t = 0 \).

We obtain for event 2 that \( y = z = 0 \), but \( x = \gamma u T \), and \( t = \gamma T \).

In Mavis’ frame, the events are at the same place, separated by a time \( T \).

In Stanley’s frame, more time (\( \gamma T \)) passes between the events, and the events happen a distance \( \gamma u T \) apart.
Lorentz Transform and Length Contraction

Is length contraction consistent with the Lorentz transform?

Consider two ends of a ruler held by Mavis on the train. The rear end is at $x_r' = 0$, and the front end at $x_f' = L_0$ (time independent),

What is the distance between the ends measured simultaneously in Stanley's frame?

$$0 = x_r' = \gamma (x_r - ut) \quad \text{(position of rear end at any given time t)}$$

$$L_0 = x_f' = \gamma (x_f - ut) \quad \text{(position of front end at time t)}$$

Take difference: $L_0 = \gamma (x_f - x_r)$

The Length measured by Stanley is

$L = x_f - x_r = L_0/\gamma$ (measure both ends at the same time).
In the train frame, the light source has frequency $f_0$ and period $T_0$. In Stanley's frame the waves are emitted a time $T = \gamma T_0$ apart (time dilation).

But what is the time interval at which they are received by Stanley?

From the diagram, we see $\lambda = cT - uT = T(c - u)$.

The frequency at which the wave crests reach Stanley is

$$f = \frac{c}{\lambda} = \frac{c}{T(c - u)} = \frac{1}{\gamma T_0(1 - \beta)}$$

$$= f_0 \sqrt{\frac{1 - \beta^2}{1 - \beta}} = f_0 \sqrt{\frac{1 + \beta}{1 - \beta}} = f_0 \sqrt{\frac{c + u}{c - u}}$$
Doppler Effect

Stanley perceives an increased frequency given by:

\[ f = f_0 \sqrt{\frac{c+u}{c-u}} \]  
(approaching source: “blue shift”)

If the source were receding from the observer at speed \( u \):

\[ f = f_0 \sqrt{\frac{c-u}{c+u}} \]  
(receding source: “red shift”)
Doppler Effect for Space Twins

space trip starts at birth
\[ u = 0.6 \, c \]
\[ \gamma = 1.25 \]

Yearly birthday greetings arrive at 2 year intervals (red shift), or at \( \frac{1}{2} \) year intervals (blue shift)
You are moving toward a mirror at speed $u$. You shine a laser beam into the mirror with light frequency $f$. You observe the reflected light to have frequency

\[ a) \quad f' = f \]
\[ b) \quad f' = f \sqrt{\frac{c+u}{c-u}} \]
\[ c) \quad f' = f \sqrt{\frac{c-u}{c+u}} \]
\[ d) \quad f' = f \frac{c+u}{c-u} \]
\[ e) \quad f' = f \frac{c-u}{c+u} \]
Note that in the Doppler effect formulas, only the relative velocity between source and observer matters (as it must be according to the principle of relativity).

This is different from the Doppler effect for sound waves, where we must distinguish between moving source/stationary observer and vice versa.
Why does the principle of relativity (i.e., there is no “special” frame of reference) not apply to sound waves?

a) Sound waves travel much slower than the speed of light.
b) Sound has nothing to do with relativity.
c) Sound waves have by their very nature a special frame of reference: The medium (air) they travel in. This allows us to decide who is moving and who is stationary.
d) The principle of relativity applies, and the Doppler shift formula derived for light must also apply to sound.
e) The formulas for sound also apply to light.
Applications of Doppler Effect

- In Astronomy, find out how celestial objects move with respect to us.
- Can also measure the temperature of light-emitting or light-absorbing materials (gases or plasma).

- In Meteorology: Find out how the rain clouds move (identify tornadoes before they touch down!) "Doppler radar".

- Law enforcement: catch speeders with "radar gun".

- Medicine: Measure how fast blood is flowing inside body (use Doppler ultrasound, record "echocardiogram").
Applications of Doppler Effect

sonogram movie of beating heart

Doppler echocardiogram movie