Physics 228 Today:

Relativity:
Force, Energy, Momentum

Photoelectric Effect:
Photons, Work Function
Momentum and Force

Momentum $p = mv$, in special relativity becomes $p = \gamma mv$.

Newton’s second law applies to this relativistic momentum:
For a force in the direction of motion:
$F_{\parallel} = \frac{dp}{dt} = \gamma^3 ma_{\parallel}$

Now consider a force, such as the magnetic force, that is perpendicular to the velocity and does not change the speed. The factor of $\gamma$ is unchanged by the magnetic force:
$F_{\perp} = \frac{dp}{dt} = \gamma ma_{\perp}$

The inertia of an object is greater than the rest mass.
Moreover, the inertia depends on the angle between the force and the velocity!
The force and acceleration vectors are not parallel!
Kinetic Energy, Total Energy, Rest Energy

• The work done accelerating an object can be calculated as the integral of force over distance: \( W = \int F \cdot dx \).

• According to the work-energy theorem, the result is just the kinetic energy of the object. The kinetic energy is in this way calculated to be \( K = \gamma mc^2 - mc^2 \).

• \( K = E - E_0 \)

• \( E = \gamma mc^2 \) is interpreted as the “total energy” of the object.

• \( E_0 = mc^2 \) is called the “rest energy” of the object.
Relativistic Mass and Rest Mass

• Kinetic energy \( K = (\gamma - 1) mc^2 = E - E_0 \)

• Corresponding to total energy \( E \) we have the “relativistic mass” \( m_{\text{rel}} = \gamma m \) (velocity dependent): \( E = m_{\text{rel}} c^2 \).

• Corresponding to rest energy we have the “rest mass” \( m \) or \( m_0 \) (does not depend on velocity): \( E_0 = mc^2 \). When we talk about the mass of a particle, we usually mean its rest mass.

• We can think of relativistic mass and (total) energy being essentially the same thing, just measured in different units (with unit conversion factor \( c^2 \)).

• Do not confuse \( m \) and \( m_{\text{rel}} \)! For example, the rest mass of a photon is zero, but the relativistic mass is \( hf/c^2 \). The photon’s momentum is \( hf/c \).
You place a very large number of relativistic (fast) particles into a box, where they bounce around, without slowing down. You then put the box on a scale. What do you measure as the mass of the particles?

a) Their rest mass $m$.

b) Their relativistic mass $m_{\text{rel}} = \gamma m$.

c) Zero, since the particles are not in contact with the walls of the box at most times.

d) Something else.
You are given an electric battery and a very accurate scale. You measure the battery’s mass before and after discharging the battery. (During discharge, the same number of electrons that leave the battery at the negative terminal re-enter at the positive terminal.) The mass after discharge will be

a) Greater than before discharge.
b) The same as before discharge.
c) Less than before discharge.
d) It depends on the observer.
e) Electricity has no mass.
Recall a previous clicker question: A torpedo moving at 0.8 c to the right explodes, sending out a flash of light in all directions. (The entire mass of the torpedo is converted to light.) Since the speed of light is independent of the speed of the source, the light shell expands around the point of explosion, in all directions in the same way:

Yet, from momentum conservation we expect the center of mass of the expanding light shell to be traveling at 0.8 c to the right. What gives?

a) The picture is incorrect: The light sphere actually moves forward as it expands.
b) The center of mass of the light is stationary, with zero total momentum.
c) Since photons have no mass, center of mass is not defined, and there is no momentum.
d) Due to the Doppler effect, the center of mass moves in the forward direction, while the geometric center is stationary.
e) Picture is incorrect: Due to length contraction, the light wave is actually an expanding ellipsoid (spheroid).
Energy-Momentum Four-Vector

Since $E = \gamma mc^2$ and $p = \gamma mv$, it follows straightforwardly that

$$ E^2 - p^2c^2 = (mc^2)^2. $$

Note that the right-hand side does not depend on velocity. It is a Lorentz invariant.

Recall the space-time 4-vector $(ct, x, y, z)$ with squared magnitude

$$ s^2 = (ct)^2 - (x^2 + y^2 + z^2). $$

Analogously, we may interpret the rest energy $mc^2$ as the magnitude of a four-vector constructed from the energy (one time-like component) and the momentum (three space-like components):

$$ p_{4V} = (E, px_c, py_c, pz_c) = (E, pc) $$

with squared magnitude $|p_{4V}|^2 = E^2 - p^2c^2 = (mc^2)^2$. It transforms from one frame to another according to the Lorentz transformation.
How can $K = (\gamma - 1) mc^2$ be consistent with $K = \frac{1}{2}mv^2$?

a) They are not.

b) The expression for $K$ is right in the non-moving frame, the other one is right in the moving frame.

c) Answer b is the right idea, but it’s got the frames backward.

d) They are both right, but the first one is for objects with large mass, while the second one is for objects with small mass.

e) The first one is right, but when $v$ is small it approaches the second one.
Review: The Wave Nature of Light

We have earlier talked about various phenomena that show the wave nature of light:

• Solving Maxwell's equations we find a traveling wave.

• Light passing through a slit diffracts like a wave, rather than having straight line trajectories like a particle.

• Light from two in-phase sources or a double slit or the reflection from top and bottom of a thin film interferes to produce maxima and minima.
Particle Nature of Light: Photoelectric Effect

(a) Light causes the cathode to emit electrons, which are pushed toward the anode by the electric-field force.

(b) Overhead view with $\vec{E}$ field reversed. Even when the direction of $\vec{E}$ field is reversed so that the electric-field force points away from the anode, some electrons still reach the anode ...

... unless the reversed potential difference has an absolute value of at least $V_0$. This stopping potential gives zero current.
Photoelectric Effect: Experiment

Same frequency, different intensities:

- The current is zero below a threshold voltage, independent of the intensity.

- The threshold voltage is called the “stopping potential”, as this voltage will stop the electrons’ motion and push them back toward the cathode.

- Since the electron’s kinetic energy is converted to potential energy during deceleration, the stopping potential is a measure of the initial kinetic energy.

Online simulation
Photoelectric Effect: Experiment

I-V graphs for different light frequencies:

- There is a threshold frequency: no electrons are emitted when the frequency is less than some threshold.
- Once the threshold frequency is exceeded, the electrons are emitted from the material. If you change the source to higher frequency, the electrons are emitted from the material with more energy.
Photoelectric Effect: Explanation

From photoemission experiments, we conclude:

- Light consists of “lumps” of energy.
- The energy in each “lump” is proportional to frequency.
- Each “lump” can be emitted or absorbed all at once or not at all.
- These “lumps” thus behave as particles of light.

The “lumps” are more technically referred to as “photons”.
Which statement is most precise?

a) Light is a wave.
b) Light is a type of particle.
\[\textbf{c) Light has both wave and particle characteristics.}\]
d) Light is a wave or a particle depending on how it is produced.
e) Light is a particle for short wavelengths, like x-rays, but a wave for long wavelength lengths, like radio waves.
Energy and Momentum of Photons

From the photoemission experiment we know that the energy of the photon is proportional to the frequency.

The proportionality constant is called “Planck’s constant”, denoted \( h \):

\[
E = hf = \frac{hc}{\lambda}.
\]

From the slope we obtain \( h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \).

What is the rest mass of a photon? Apply \( E = \gamma mc^2 \) to photons (\( v = c \)):

\[
m = \frac{E}{\gamma c^2} = \frac{E}{c^2} \sqrt{1 - \left(\frac{v}{c}\right)^2} = 0
\]

For a particle with zero rest mass the energy momentum relationship \( E^2 - p^2c^2 = (mc^2)^2 \) becomes \( E = pc \).

The momentum of the photon is then \( p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \).
If we apply a potential $V_0$ that just stops all the emitted electrons, we conclude that the maximum kinetic energy of the emitted electrons is:

$$K_{\text{max}} = -eV_0.$$  

The experiment shows that the maximum kinetic energy is just the photon energy minus some offset $\phi$:

$$K_{\text{max}} = E - \phi = hf - \phi.$$  

The downward shift of the curve $\phi$ is called the "work function" (not really a function but a constant) of the material from which the electrons are being emitted. It is typically a few electron-Volts (eV).
The work function depends on the material of the cathode:

- It represents the amount of work a photon has to do to remove a single electron from the cathode. The leftover photon energy is converted into kinetic energy of the electron.

- The following table shows the work function for different metals:

<table>
<thead>
<tr>
<th>METAL</th>
<th>WORK FUNCTION ($W_0$) in J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>$6.54 \times 10^{-19}$</td>
</tr>
<tr>
<td>Zinc</td>
<td>$6.89 \times 10^{-19}$</td>
</tr>
<tr>
<td>Silver</td>
<td>$7.58 \times 10^{-19}$</td>
</tr>
</tbody>
</table>
Applications of Photoelectric Effect

- Photomultiplier tubes: ultra-sensitive light detectors, can detect single photons.
- Electron spectroscopy: Learn about the electron energy levels in different materials.
- Night vision cameras and night vision scopes.