More Relativity:

Space-Time Diagrams
Lorentz Transformations
Twin Paradox in Detail
Ladder in Garage Paradox
It the primed frame moves at velocity \( u \) in the \( x \)-direction, in non-relativistic physics the coordinates of an event in the two frames are related by:

\[
\begin{align*}
    x' &= x - ut \\
    y' &= y \\
    z' &= z \\
    t' &= t
\end{align*}
\]

\[
\begin{align*}
    x &= x' + ut \\
    y &= y' \\
    z &= z' \\
    t &= t'
\end{align*}
\]

Valid for small velocities \( u \) only!
Lorentz Transformation

In special relativity, the coordinate transformation becomes:

\[ \begin{align*}
x' &= \gamma (x - u t) \\
y' &= y \\
z' &= z \\
t' &= \gamma (t - u x/c^2)
\end{align*} \]

\[ \begin{align*}
x &= \gamma (x' + u t') \\
y &= y' \\
z &= z' \\
t &= \gamma (t' + u x'/c^2)
\end{align*} \]

Frame \( S' \) moves relative to frame \( S \) with constant velocity \( u \) along the common \( x-x' \)-axis.

Origins \( O \) and \( O' \) coincide at time \( t = 0 = t' \).

The Lorentz coordinate transformation relates the spacetime coordinates of an event as measured in the two frames: \((x, y, z, t)\) in frame \( S \) and \((x', y', z', t')\) in frame \( S' \).
Space-Time Diagrams

1. Simplified Space-Time Diagram for Particle Moving at Constant Speed

2. Simplified Space-Time Diagram Normalizes Axis Scales to Seconds and Light-Seconds

3. Space-Time Diagram showing Possible World Lines

4. Light Cone With Two Space Dimensions

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Clicker Question

Which spacetime diagram is most appropriate for an ideal pendulum?

A)

B)

C)

D)
Starting from event 1, which other events could be reached by travel?

a) A
b) B  Requires faster-than-light travel
c) C  Requires traveling into the past
d) More than one
e) None
Space-Time Diagram of Moving Frame
(Galilei Transformation)

\[
x' = x - u \cdot t
\]
\[
y' = y
\]
\[
z' = z
\]
\[
t' = t
\]

\[
x = x' + u \cdot t'
\]
\[
y = y'
\]
\[
z = z'
\]
\[
t = t'
\]
Space-Time Diagram of Moving Frame (Lorentz Transformation)

\[ x' = \gamma (x - u \, t) \]
\[ y' = y \]
\[ z' = z \]
\[ t' = \gamma (t - \frac{u \, x}{c^2}) \]

\[ x = \gamma (x' + u \, \gamma \, t') \]
\[ y = y' \]
\[ z = z' \]
\[ t = \gamma (t' + \frac{u \, x'}{c^2}) \]

Mavis moving at \( u = 0.6 \, c \)

(lines of simultaneity are horizontal in Stanley's frame)
Clocks in Space

- To simplify the discussion, we are going to pretend that space is filled with an infinite array of clocks, all synchronized with each other.
- Since time is relative to the observer, we need two sets of clocks: One fixed to Stanley’s frame, and one traveling with Mavis in her frame.
- As we have seen, the two sets of clocks will show different times.
- In particular, Stanley will observe Mavis’s clocks as not synchronized, and vice versa.
- In this way, any “event” can be assigned definite x, y, z, and t coordinates, as well as definite x’, y’, z’, and t’ coordinates.

The grid is three dimensional; identical planes of clocks lie in front of and behind the page, connected by grid lines perpendicular to the page.
Space-Time Diagrams of Clocks in Space

Mavis's clocks

Stanley's clocks

Mavis: No, they're not!

Stanley: All of my clocks are synchronized.

Mavis: All of MY clocks are synchronized.
Which line could represent a set of synchronized clocks in the spaceship frame?

a) AB
b) CD
c) EF
From Stanley’s point of view, Mavis’s clock runs slow. From Mavis’s point of view, Stanley’s clock runs slow.
Mark Kelly has been staying on Earth.

Scott Kelly has been in orbit for the last year. When he returns (tomorrow!), he will be 9 milliseconds younger!
Twin Paradox Revisited

Al

Bert

stationary twin

traveling twin

simultaneity planes (ret. trip)

simultaneity planes (trip out)

Al

Bert
Twin Paradox Revisited

Space trip starts at birth
\[ u = 0.6 \, \text{c} \]
\[ \gamma = 1.25 \]

World line of space twin

World line of earth twin

Lines of simultaneity before turnaround

Lines of simultaneity after turnaround

Time (years)

Distance (lightyears)

Age of earth twin

Age of space twin

Simultaneity Gap
Space-Time Diagram from Space Twin’s Point of View

Lorentz transformed space-time diagram using space twin’s inertial frame (outbound trip) as the rest frame.
Signaling Between Twins

Space trip starts at birth.

\[ u = 0.6 \, c \]

\[ \gamma = 1.25 \]

Each year on his birthday, each twin sends the other a snapchat greeting.

Birthday greetings travel at the speed of light (45 degree diagonal lines).
Ladder in Garage Paradox

Ladder won’t fit into garage! What to do?

If the ladder is shoved in fast enough, it will (momentarily) fit due to length contraction!

Yet, from the point of view of the ladder, the garage is moving and is contracted. So the ladder doesn’t fit after all! What gives?
a) The ladder fits in the garage due to ladder’s length contraction.

b) The ladder does not fit in the garage because of contraction of garage.

c) Ladders will never move fast enough for this to work.

d) The ladder fits into the garage as seen in the garage’s frame of reference, but not in ladder’s frame.

e) This is a logical contradiction that disproves special relativity.
Ladder in Garage Paradox

Stationary garage, moving ladder:
Doors close simultaneously!

Stationary ladder, moving garage:
Doors close one after the other!
When constructing this space-time diagram, how would we find Mavis's world line?

a) By setting $x = 0$ in the LT.
b) By setting $t = 0$ in the LT.
c) By setting $x' = 0$ in the LT.
d) By setting $t' = 0$ in the LT.
e) None of the above.