Free Particle Wave Equation

**Description:** Solve the time-independent Schrödinger equation for a free particle and find an expression for the energy of the particle.

The Schrödinger equation for a free particle (no potential energy) is

\[ -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi. \]

**Part A**

What is the most general solution \( \psi \) of the time-independent Schrödinger equation?

**Hint 1. Testing a possible answer**

The best way to test a possible solution is to plug it into the equation. If the solution is correct, the \( x \) dependence should cancel out, since the solution has to be valid for all values of \( x \). Keep in mind that you have to find the most general form of the solution.

**ANSWER:**

- \( A \sin(kx) \)
- \( A \sin(kx) + B \cos(kx) \)
- \( Ae^{-kx} + Be^{kx} \)

**Part B**

Express the energy \( E \) of the particle in terms of the wave number \( k \) of the particle.

Express your answer in terms of wave number \( k \), mass \( m \), and Planck's constant divided by \( 2\pi \): \( \hbar \).

**ANSWER:**

\[ E = \frac{k^2 \hbar^2}{2m} \]

**Part C**

To normalize this wave function, you must calculate the integral \( \int_{-\infty}^{\infty} |\psi|^2 dx \). What is the value of this integral?

**ANSWER:**

- \( A^2 + B^2 \)
- \( 2AB \)
- \( A^2 + 2AB + B^2 \)
- 0
- \( \infty \)

Because this integral does not converge, a free particle wave function is unnormalizable. This is due to the fact that a free particle wave function has no boundaries and thus is unlocalized. This means that there is the same probability of finding a particle anywhere in the universe.
Schrödinger Equation and the Particle in a Box

Description: A solution to the Schrödinger equation is checked. The values of constants are then determined from normalization conditions.

Learning Goal:
To become familiar with the Schrödinger equation and its solution for the simple case of the particle in a box.

The most important equation in quantum mechanics is the Schrödinger equation,

\[ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi + U(x) \psi = E \psi, \]

where \( \hbar \) is Planck's constant divided by \( 2\pi \) (i.e., \( \hbar = h/(2\pi) \)).

Given a potential energy function \( U(x) \), solving the Schrödinger equation allows you to determine the particle wave functions. Finding solutions to the Schrödinger equation, for most potentials, is beyond the scope of introductory physics. However, you are able to check a solution, once it is presented to you. You will do this for the simple case of the particle in a box.

The quantum mechanical particle in a box has a particularly simple potential energy function. Although it does have some real-world applications, the particle in a box is also important as an illustration of many key concepts from quantum mechanics.

Consider a particle in a potential well with infinitely high walls. The potential energy function is formally written as

\[ U(x) = \begin{cases} 0, & 0 < x < L, \\ \infty, & x < 0 \text{ or } x > L, \end{cases} \]

where \( L \) is the width of the box. It is claimed that each of the functions

\[ \psi_n(x) = \begin{cases} C \sin(n\pi x/L), & 0 \leq x \leq L, \\ 0, & x < 0 \text{ or } x > L, \end{cases} \]

for \( n = 1, 2, 3 \ldots \) is a solution to the Schrödinger equation for the particle in a box. You will prove this and calculate the proper value for \( C \).

By inspection, you should be able to see that \( \psi = 0 \) is a mathematical solution to any Schrödinger equation, so the functions \( \psi_n(x) \) are clearly valid solutions outside the interval \( 0 \leq x \leq L \).

Part A

Consider the interval \( 0 \leq x \leq L \). What is the second derivative, with respect to \( x \), of the wave function \( \psi_n(x) \) in this interval?

Express your answer in terms of \( n, x, L, \) and \( C \).

**ANSWER:**

\[ \frac{d^2}{dx^2} \psi_n(x) = -\frac{n^2 \pi^2}{L^2} C \sin \left( \frac{n\pi x}{L} \right) \]

Part B

What is \( U(x)\psi_n(x) \) in the interval \( 0 \leq x \leq L \)?

Express your answer in terms of \( n, L, \) and \( C \).

**ANSWER:**

\[ U(x)\psi_n(x) = 0 \]

Part C

\( E \) is an as yet undetermined constant: the energy of the particle. What is \( E\psi_n(x) \) in the interval \( 0 \leq x \leq L \)?

Express your answer in terms of \( n, L, E, \) and \( C \).

**ANSWER:**

\[ E\psi_n(x) = EC\sin \left( \frac{n\pi x}{L} \right) \]

Part D

Combine your answers from Parts A and B. Find the expression for the left side of the Schrödinger equation valid on the interval \( 0 \leq x \leq L \).

Express your answer in terms of \( \hbar, m, n, x, L, \) and \( C \).

**ANSWER:**

\[ \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} \psi_n(x) + U(x)\psi_n(x) = E\psi_n(x) \]

\[ \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} C \sin \left( \frac{n\pi x}{L} \right) + 0 = EC\sin \left( \frac{n\pi x}{L} \right) \]

\[ \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} C \sin \left( \frac{n\pi x}{L} \right) = EC\sin \left( \frac{n\pi x}{L} \right) \]

\[ E = \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} \]

\[ \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} C \sin \left( \frac{n\pi x}{L} \right) = EC\sin \left( \frac{n\pi x}{L} \right) \]

\[ \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} C \sin \left( \frac{n\pi x}{L} \right) = EC\sin \left( \frac{n\pi x}{L} \right) \]

\[ E = \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} \]

\[ \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} C \sin \left( \frac{n\pi x}{L} \right) = EC\sin \left( \frac{n\pi x}{L} \right) \]

\[ \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} C \sin \left( \frac{n\pi x}{L} \right) = EC\sin \left( \frac{n\pi x}{L} \right) \]

\[ E = \frac{-\hbar^2}{2m} \frac{n^2 \pi^2}{L^2} \]
This entire expression is just \( \sin\left(\frac{n\pi x}{L}\right) \) multiplied by a positive constant. Since you have already found the right side of the equation to be \( \sin\left(\frac{n\pi x}{L}\right) \) multiplied by a positive constant (|\( C |\)), you have proven that, if the two constants are equal, \( \psi_n(x) \) is a mathematical solution to the Schrödinger equation for the particle in a box. You will soon determine if it is also a physical solution.

**Part E**

Combine your answers from Parts C and D to find the value of \( E_n \), the energy of a particle with wave function \( \psi_n(x) \).

Express your answer in terms of \( n \), \( m \), \( \hbar \), and \( L \).

ANSWER:

\[
E_n = \frac{n^2\pi^2 \hbar^2}{2mL^2}
\]

Even in the simple case of the particle in a box, one of the main ideas of quantum theory—the quantization of energy—may be seen in the discrete allowed energy levels:

\[
E_1 = \frac{\hbar^2\pi^2}{2mL^2}, \quad E_2 = \frac{4\hbar^2\pi^2}{2mL^2}, \quad E_3 = \frac{9\hbar^2\pi^2}{2mL^2}, \quad \ldots
\]

In this context, you can see how the quantization of energy is a natural consequence of applying the boundary conditions to solutions of the Schrödinger equation.

**Part F**

For a solution to be a physical solution, it must satisfy several criteria. First, it must be continuous everywhere. Second, it must have a continuous derivative everywhere, except for points where the potential energy becomes infinite (as it does at the walls of the box). Finally, it must be normalizable.

In this case, you can check the first criterion by noting that the two functions \( C \sin\left(\frac{n\pi x}{L}\right) \) and 0 are continuous and that they have the same value at \( x = 0 \) and \( x = L \), where their domains meet.

To check the second criterion, simply take the first derivative of \( C \sin\left(\frac{n\pi x}{L}\right) \) and 0. Both derivatives are continuous functions in their domains. The points where the domains meet are exactly the points where the potential energy becomes infinite, so you don't have to check for continuity there.

The third criterion requires that there exist some value of \( C \) such that

\[
\int_{-\infty}^{\infty} |\psi_n(x)|^2 \, dx = 1.
\]

Since \( \psi_n(x) \) is zero outside of the interval \( 0 \leq x \leq L \), this equation reduces to

\[
\int_0^L C^2 \sin^2\left(\frac{n\pi x}{L}\right) \, dx = 1
\]

Use this equation to find the unique positive value of \( C \).

Express your answer in terms of \( L \).

**Hint 1. Evaluate the integral**

Find the value of

\[
\int_0^L C^2 \sin^2\left(\frac{n\pi x}{L}\right) \, dx.
\]

Keep in mind that \( n \) is a positive integer when you evaluate the integral.

Express your answer in terms of \( C \) and \( L \).

**Hint 1. Integrating \( \sin^2(u) \)**

To integrate \( \sin^2(u) \), use the identity \( \sin^2(u) = (1/2)[1 - \cos(2u)] \).

ANSWER:

\[
\int_0^L C^2 \sin^2\left(\frac{n\pi x}{L}\right) \, dx = \frac{C^2L}{2}
\]

Now, substitute this value into the equation for the normalization condition and solve for \( C \).

ANSWER:
Finding Probabilities from the Wave Function

Description: The wavefunctions for various states of a particle in an infinite potential well are used to find the probability that a particle is found in a particular region.

Learning Goal:
To use the wave function for a particle in a box to calculate the probability that the particle is found in various regions within the box.

The quantum mechanical probability that a particle described by the (normalized) wave function $\psi(x)$ is found in the region between $a$ and $b$ is

$$ P = \int_a^b |\psi(x)|^2 \, dx. $$

The specific example of a particle trapped in an infinitely deep potential well, sometimes called a particle in a box, serves as good practice for calculating these probabilities, because the wave functions for this situation are easy to write down. If the ends of the box are at $x = 0$ and $x = L$, then the allowed wave functions are

$$ \psi(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right), & \text{for } 0 \leq x \leq L, \\ 0, & \text{for all other } x, \end{cases} $$

where $n = 1$ is the ground-state wave function, $n = 2$ is the first excited state, etc.

Here are a few integrals that may prove useful:

- $\int \sin(kx) \, dx = -\frac{1}{k} \cos(kx) + C.$
- $\int \cos(kx) \, dx = \frac{1}{k} \sin(kx) + C.$
- $\int \sin^2(kx) \, dx = \frac{x}{2} - \frac{1}{4k} \sin(2kx) + C.$
- $\int \cos^2(kx) \, dx = \frac{x}{2} + \frac{1}{4k} \sin(2kx) + C.$

**Part A**

If the particle in the box is in the second excited state (i.e., $n = 3$), what is the probability $P$ that it is between $x = L/3$ and $x = L$? To find this probability, you will need to evaluate the integral

$$ \int_{L/3}^L \left( \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \right)^2 \, dx = \frac{2}{L} \int_{L/3}^L \sin^2 \left( \frac{n\pi x}{L} \right) \, dx. $$

Express your answer as a number between 0 and 1 to three significant figures.

**ANSWER:**

$$ P = 0.667 $$

**Part B**

If the particle is in the first excited state, what is the probability that it is between $x = 0.1L$ and $x = 0.2L$?

Express your answer as a number between 0 and 1 to three significant figures.

**Hint 1. How to set up the integral**

The particle is now in the first excited state ($n = 2$). The integral for this part is the same as the integral from Part A, except for the different value of $n$ and the different limits of integration (which are now $0.1L$ and $0.2L$).

**ANSWER:**

$$ P = 0.129 $$

**Part C**

If the particle is in the ground state, what is the probability that it is in a window $\Delta x = 0.0002L$ wide with its midpoint at $x = 0.700L$?

You should be able to answer this part without evaluating any integrals! Since $\Delta x$ is so small, you can assume that $\psi(x)$ remains constant over that interval, so the integral is approximately

$$ \Delta x \left( \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \right)^2. $$

Express your answer as a number between 0 and 1 to three significant figures.
Part D

Assume a window \( \Delta x \) as in Part C. Compared to a particle in the ground state, which of the following statements is true for a particle in the first excited state?

**Hint 1. How to approach the problem**

Since the same window is used for both states, \( \Delta x \) does not change. Therefore, whichever state has a larger value for \( \| \psi(0.700L) \|^2 \) will have the higher probability.

**ANSWER:**

- A particle in the first excited state is more likely to be found in the window \( \Delta x \).
- A particle in the first excited state is less likely to be found in the window \( \Delta x \).
- A particle in the first excited state is equally likely to be found in the window \( \Delta x \).

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**Vision and the Particle in a Box**

**Description:** The wavelength of the photon needed to excite an electron in a box is found. The result is then applied to retinal, the key light-absorbing molecule in eyes.

Though the particle in a box (infinite potential well) seems like a very unrealistic potential, it can actually be used to explain a bit about how humans see. The important light-absorbing molecule in human eyes is called retinal. Retinal consists of a chain of carbon atoms, roughly \( 1.5 \times 10^{-9} \) m long. An electron in this long chain molecule behaves very much like a particle in a box.

**Part A**

Find the wavelength \( \lambda \) of the photon that must be absorbed by an electron to move it from the \( n \)th state of a box to the \((n+1)\)th state. Assume that the box has length \( L \) and that the electron has mass \( m \).

Express your answer in terms of \( E \), \( c \), and \( \hbar \).

**Hint 1. Find the wavelength in terms of energy**

Find the wavelength \( \lambda \) of a photon in terms of the photon's energy \( E \). Recall that photons are described by the equations \( E = \hbar \omega \) and \( c = \lambda f \), where \( \hbar \) is Planck's constant divided by \( 2\pi \) and the speed of light. \( \lambda \) is the frequency, and \( \omega \) is the angular frequency.

Express your answer in terms of \( E \), \( c \), and \( \hbar \).

**Answer:**

\[
\lambda = \frac{2\pi \hbar}{E}
\]

**Hint 2. Find the energy of the photon**

Find the difference between the energy \( E_{n+1} \) of the \((n+1)\)th state for the electron and the energy \( E_n \) of the \(n\)th state.

Express your answer in terms of \( E_n \) and \( E_{n+1} \).

**Answer:**

\[
E_{n+1} - E_n = \frac{\hbar^2}{2mL^2} \left( \frac{n+1}{n} \right)^2
\]

---

Answer: \( 2.62 \times 10^{-4} \)

A particle in the first excited state is more likely to be found in the window \( \Delta x \).
Part B

The retinal molecule has 12 electrons that are free to move about the chain. For reasons that you may learn later, these 12 electrons fill the first 6 states of the box (with 2 electrons in each state). Thus, the lowest energy photon that can be absorbed by this molecule would be the one that moves an electron from the 6th state to the 7th. Use the equation that you found in Part A to determine the wavelength $\lambda$ of this photon. Use the length of the retinal molecule given in the introduction as the length of the box and use $m=9.11 \times 10^{-31}$ kg for the mass of the electron.

Express your answer in nanometers to two significant figures.

\[
\text{\text{Answer:} } \lambda \approx 570 \text{ nm}
\]

A photon with this wavelength lies in the green part of the spectrum.

Part C

In a human eye, there are three types of cones that allow us to see colors. The three different types are most sensitive to red, green, and blue light, respectively. All three contain retinal bonded to a large protein. The way that retinal bonds to the protein can change the length of the potential well within which the electrons are confined. How would the length have to change from that given in the introduction to make the molecule more sensitive to blue or red light?

\[E_{n+1} - E_n = \frac{(2n+1)^2\hbar^2}{2mL^2} = \frac{4mcL^2}{(2n+1)^2}\hbar^2\]

\[\lambda = \frac{h}{p} = \frac{h}{\sqrt{(2n+1)^2\hbar^2}} = \frac{h\sqrt{2mL^2}}{(2n+1)^2}\hbar^2 = \frac{4mcL^2\hbar}{(2n+1)^2}\hbar^2 = \frac{4mcL^2}{(2n+1)^2}\hbar^2
\]

Once retinal has absorbed a photon, it changes shape, initiating a cascade of effects that eventually creates a nerve impulse. This impulse gets fed to the brain, where it is processed along with the signals from all of the other light-sensing cells in the eye, allowing you to see.

Exercise 39.3

Description: An electron has a de Broglie wavelength of $\lambda$. (a) Determine the magnitude of its momentum. (b) Determine its kinetic energy in joules. (c) Determine its kinetic energy in electron volts.

An electron has a de Broglie wavelength of $2.78 \times 10^{-10}$ m.

Part A

Determine the magnitude of its momentum.

\[p = \frac{\hbar}{\lambda} = \frac{6.63 \times 10^{-34}}{2.78 \times 10^{-10}} = 2.36 \times 10^{-24} \text{ kg m/s}
\]

\[E_{\text{kin}} = \frac{p^2}{2m} = \frac{(2.36 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31}} = 2.69 \times 10^{-19} \text{ J}
\]

\[E_{\text{kin}} = \frac{p^2}{2m} = \frac{(2.36 \times 10^{-24})^2}{9.11 \times 10^{-31}} = 2.69 \times 10^{-19} \text{ eV}
\]
Exercise 39.12

Description: A beam of electrons is accelerated from rest through a potential difference of \( V \) and then passes through a thin slit. When viewed far from the slit, the diffracted beam shows its first diffraction minima at \( \pm \theta \) from the original direction of the beam.

A beam of electrons is accelerated from rest through a potential difference of 0.200 \( \text{keV} \) and then passes through a thin slit. When viewed far from the slit, the diffracted beam shows its first diffraction minima at \( \pm 13.6 \) \( ^\circ \) from the original direction of the beam.

Part A

Do we need to use relativity formulas? Select the correct answer and explanation.

ANSWER:

Yes. The electrons gain kinetic energy \( K \) as they are accelerated through a potential difference \( V \), so \( Ve=K=mc^2/\gamma-1 \). The potential difference is 0.200 \( \text{keV} \), so \( Ve=0.200 \text{ \text{keV}} \). Solving for \( \gamma \) and using the fact that the rest energy of an electron is 0.511 \( \text{MeV} \), we have \( \gamma - 1 = (0.511 \text{MeV})/(0.200 \text{keV}) \) so \( \gamma \gg 1 \) which means that we have to use special relativity.

No. The electrons gain kinetic energy \( K \) as they are accelerated through a potential difference \( V \), so \( Ve=K=mc^2/\gamma-1 \). The potential difference is 0.200 \( \text{keV} \), so \( Ve=0.200 \text{ \text{keV}} \). Solving for \( \gamma \) and using the fact that the rest energy of an electron is 0.511 \( \text{MeV} \), we have \( \gamma - 1 = (0.511 \text{MeV})/(0.200 \text{keV}) \) so \( \gamma - 1 >> 1 \) which means that we do not have to use special relativity.

Yes. The electrons gain kinetic energy \( K \) as they are accelerated through a potential difference \( V \), so \( Ve=K=mc^2/\gamma-1 \). The potential difference is 0.200 \( \text{keV} \), so \( Ve=0.200 \text{ \text{keV}} \). Solving for \( \gamma \) and using the fact that the rest energy of an electron is 0.511 \( \text{MeV} \), we have \( \gamma - 1 = (0.511 \text{MeV})/(0.200 \text{keV}) \) so \( \gamma - 1 << 1 \) which means that we do not have to use special relativity.

No. The electrons gain kinetic energy \( K \) as they are accelerated through a potential difference \( V \), so \( Ve=K=mc^2/\gamma-1 \). The potential difference is 0.200 \( \text{keV} \), so \( Ve=0.200 \text{ \text{keV}} \). Solving for \( \gamma \) and using the fact that the rest energy of an electron is 0.511 \( \text{MeV} \), we have \( \gamma - 1 = (0.511 \text{MeV})/(0.200 \text{keV}) \) so \( \gamma - 1 \ll 1 \) which means that we do not have to use special relativity.

Part B

How wide is the slit?

Express your answer with the appropriate units.

ANSWER:

Yes, the electrons gain kinetic energy \( K \) as they are accelerated through a potential difference \( V \), so \( Ve=K=mc^2/\gamma-1 \). The potential difference is 0.200 \( \text{keV} \), so \( Ve=0.200 \text{ \text{keV}} \). Solving for \( \gamma \) and using the fact that the rest energy of an electron is 0.511 \( \text{MeV} \), we have \( \gamma - 1 = (0.511 \text{MeV})/(0.200 \text{keV}) \) so \( \gamma - 1 \ll 1 \) which means that we do not have to use special relativity.

No, the electrons gain kinetic energy \( K \) as they are accelerated through a potential difference \( V \), so \( Ve=K=mc^2/\gamma-1 \). The potential difference is 0.200 \( \text{keV} \), so \( Ve=0.200 \text{ \text{keV}} \). Solving for \( \gamma \) and using the fact that the rest energy of an electron is 0.511 \( \text{MeV} \), we have \( \gamma - 1 = (0.511 \text{MeV})/(0.200 \text{keV}) \) so \( \gamma - 1 \gg 1 \) which means that we have to use special relativity.
Exercise 39.45

**Description:** The uncertainty in the $y$-component of a proton's position is $\Delta y$. (a) What is the minimum uncertainty in a simultaneous measurement of the $y$-component of the proton's velocity? (b) The uncertainty in the $z$-component of an electron's velocity...

The uncertainty in the $y$-component of a proton's position is $1.9 \times 10^{-12} \text{ m}$. 

**Part A**

What is the minimum uncertainty in a simultaneous measurement of the $y$-component of the proton's velocity?

**ANSWER:**

$$\Delta v_y = \frac{\hbar}{2 \pi \Delta y} = 1.7 \times 10^4 \text{ m/s}$$

Also accepted: $\frac{\hbar}{(\Delta y)^2 2 \pi 1.67 \times 10^{-27}} = 1.7 \times 10^4$, $\frac{6.63 \times 10^{-34}}{(\Delta y)^2 2 \pi 1.67 \times 10^{-27}} = 1.7 \times 10^4$, $\frac{6.63 \times 10^{-34}}{(\Delta y)^2 2 \pi 1.673 \times 10^{-27}} = 1.7 \times 10^4$, $\frac{1.055 \times 10^{-34}}{2 \pi \Delta y 1.67 \times 10^{-27}} = 1.7 \times 10^4$, $\frac{1.055 \times 10^{-34}}{2 \pi \Delta y 1.673 \times 10^{-27}} = 1.7 \times 10^4$, $\frac{1.055 \times 10^{-34}}{2 \pi \Delta y 1.67 \times 10^{-27}} = 1.7 \times 10^4$.

**Part B**

The uncertainty in the $z$-component of an electron's velocity is $0.280 \text{ m/s}$. What is the minimum uncertainty in a simultaneous measurement of the $z$-coordinate of the electron?

**ANSWER:**

$$\Delta z = \frac{\hbar}{2 \pi \Delta v_z} = 2.07 \times 10^{-4} \text{ m}$$

Also accepted: $\frac{\hbar}{2 \pi \Delta v_z 9.109 \times 10^{-31}} = 2.07 \times 10^{-4}$, $\frac{6.63 \times 10^{-34}}{2 \pi \Delta v_z 9.109 \times 10^{-31}} = 2.07 \times 10^{-4}$, $\frac{6.63 \times 10^{-34}}{2 \pi \Delta v_z 9.11 \times 10^{-31}} = 2.07 \times 10^{-4}$, $\frac{1.055 \times 10^{-34}}{2 \Delta v_z 9.109 \times 10^{-31}} = 2.07 \times 10^{-4}$, $\frac{1.055 \times 10^{-34}}{2 \Delta v_z 9.11 \times 10^{-31}} = 2.07 \times 10^{-4}$.

Problem 39.73

**Description:** The radii of atomic nuclei are of the order of $5.0 \times 10^{-15} \text{ m}$. (a) Estimate the minimum uncertainty in the momentum of an electron if it is confined within a nucleus. (b) Take this uncertainty in momentum to be an estimate of the magnitude of the...

The radii of atomic nuclei are of the order of $5.0 \times 10^{-15} \text{ m}$.

**Part A**

Estimate the minimum uncertainty in the momentum of an electron if it is confined within a nucleus.

**ANSWER:**

- $1.3 \times 10^{-25} \text{ kg \cdot m/s}$
- $2.4 \times 10^{-22} \text{ kg \cdot m/s}$
- $2.0 \times 10^{-17} \text{ kg \cdot m/s}$
- $1.1 \times 10^{-20} \text{ kg \cdot m/s}$

**Part B**

Take this uncertainty in momentum to be an estimate of the magnitude of the momentum. Use the relativistic relationship between energy and momentum, equation $E^2 = (mc^2)^2 + (pc)^2$, to obtain an estimate of the kinetic energy of an electron confined within a nucleus.

**ANSWER:**

$$1.3 \times 10^{-25} \text{ kg \cdot m/s} \sqrt{(1.66 \times 10^{-25} \text{ kg \cdot m/s})^2 + (1.7 \times 10^4 \text{ m/s})^2} = 2.0 \times 10^{-17} \text{ J}$$

$$2.4 \times 10^{-22} \text{ kg \cdot m/s} \sqrt{(1.66 \times 10^{-22} \text{ kg \cdot m/s})^2 + (1.7 \times 10^4 \text{ m/s})^2} = 2.0 \times 10^{-17} \text{ J}$$

$$2.0 \times 10^{-17} \text{ kg \cdot m/s} \sqrt{(1.66 \times 10^{-17} \text{ kg \cdot m/s})^2 + (1.7 \times 10^4 \text{ m/s})^2} = 2.0 \times 10^{-17} \text{ J}$$

$$1.1 \times 10^{-20} \text{ kg \cdot m/s} \sqrt{(1.66 \times 10^{-20} \text{ kg \cdot m/s})^2 + (1.7 \times 10^4 \text{ m/s})^2} = 2.0 \times 10^{-17} \text{ J}$$
Part C
Calculate the magnitude of the Coulomb potential energy of a proton and an electron separated by \( r \).

**ANSWER:**

\(|U| = 0.29 \text{ MeV}\)

Also accepted: 0.287, 0.29, 0.288, 0.29

Part D
Compare the energies calculated in parts \( \text{B} \) and \( \text{C} \).

**ANSWER:**

\[ \frac{K}{|U|} = 67 \]

Also accepted: 67, 67.0, 67.0, 70, 70.0, 70.0, 67

Part E
On the basis of the result of part \( \text{D} \), could there be electrons within the nucleus?

**ANSWER:**

- yes
- no

Exercise 39.19
**Description:** A hydrogen atom is in a state with energy \( E \). (a) In the Bohr model, what is the angular momentum of the electron in the atom, with respect to an axis at the nucleus?

A hydrogen atom is in a state with energy \(-0.378 \text{ eV}\).

Part A
In the Bohr model, what is the angular momentum of the electron in the atom, with respect to an axis at the nucleus?

**Express your answer using three significant figures.**

**ANSWER:**

\[ L = n \times 1.055 \times 10^{-34} \text{ kg m}^2/\text{s} \]

Also accepted: \( n \times 1.05 \times 10^{-34} \), \( n \times 6.63 \times 10^{-34}/(2\pi) \), \( n \times 1.055 \times 10^{-34} \)

Exercise 39.6
**Description:** A nonrelativistic free particle with mass \( m \) has kinetic energy \( K \). (a) Derive an expression for the de Broglie wavelength of the particle in terms of \( m \) and \( K \).

(b) What is the de Broglie wavelength of an 800-eV electron?

A nonrelativistic free particle with mass \( m \) has kinetic energy \( K \).

Part A
Derive an expression for the de Broglie wavelength of the particle in terms of \( m \) and \( K \).

**Express your answer in terms of \( \text{texp}(m) \), \( \text{texp}(K) \), and the Planck's constant \( \text{texp}(h) \).**

**ANSWER:**

\[ \lambda = \frac{h}{\sqrt{2Km}} \]
What is the de Broglie wavelength of an 800-eV electron?

Express your answer using one significant figure.

ANSWER: \[ \lambda = 4.34 \times 10^{-11} \text{ m} \]

Also accepted: \[ 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11}, 4 \times 10^{-11} \]

Exercise 39.32

Description: Pulsed dye lasers emit light of wavelength \( \lambda \) nm in \( t \) ms pulses to remove skin blemishes such as birthmarks. The beam is usually focused onto a circular spot \( d \) mm in diameter. Suppose that the output of one such laser is \( W \) W.

Pulsed dye lasers emit light of wavelength 585 nm in 0.45 ms pulses to remove skin blemishes such as birthmarks. The beam is usually focused onto a circular spot 4.6 mm in diameter. Suppose that the output of one such laser is 20.0 W.

Part A

What is the energy of each photon, in eV?

ANSWER: \[ E_{\text{photon}} = 2.12 \text{ eV} \]

Part B

How many photons per square millimeter are delivered to the blemish during each pulse?

ANSWER:

\[
 n = \frac{Pt(6.626 \times 10^{-34} \text{c} / \lambda m \times 10^{-9})}{(\pi d/2)^2} = 1.6 \times 10^{15} \text{ photons/(mm)^2}
\]

Also accepted: \[
 1.59 \times 10^{15}, \frac{Pt(6.63 \times 10^{-34} \text{c} / \lambda m \times 10^{-9})}{(\pi d/2)^2} = 1.6 \times 10^{15}, \frac{Pt(6.63 \times 10^{-34} \text{c} / \lambda m \times 10^{-9})}{(\pi d/2)^2} = 1.6 \times 10^{15}, \frac{Pt(6.63 \times 10^{-34} \text{c} / \lambda m \times 10^{-9})}{(\pi d/2)^2} = 1.6 \times 10^{15}
\]

Exercise 39.8

Description: (a) What is the de Broglie wavelength for an electron with speed \( v_1 \)? (Hint: Use the correct relativistic expression for linear momentum if necessary). (b) What is the de Broglie wavelength for an electron with speed \( v_2 \)? (Hint: Use the...
\[ \lambda_2 = 6.626 \times 10^{-34}/(9.109 \times 10^{-31} \times 3.00 \times 10^8)/(1/(1/v_2^2-1))^{0.5} = 8.80 \times 10^{-13} \text{ (m m)} \]

Also accepted: 
- \[ 6.63 \times 10^{-34}/(9.109 \times 10^{-31} \times 3.00 \times 10^8)/(1/(1/v_2^2-1))^{0.5} = 8.81 \times 10^{-13} \]
- \[ 6.63 \times 10^{-34}/(9.11 \times 10^{-31} \times 2.998 \times 10^8)/(1/(1/v_2^2-1))^{0.5} = 8.81 \times 10^{-13} \]
- \[ 6.63 \times 10^{-34}/(9.11 \times 10^{-31} \times 3.00 \times 10^8)/(1/(1/v_2^2-1))^{0.5} = 8.81 \times 10^{-13} \]
- \[ 6.626 \times 10^{-34}/(9.109 \times 10^{-31} \times 3.00 \times 10^8)/(1/(1/v_2^2-1))^{0.5} = 8.80 \times 10^{-13} \]
- \[ 6.626 \times 10^{-34}/(9.11 \times 10^{-31} \times 2.998 \times 10^8)/(1/(1/v_2^2-1))^{0.5} = 8.81 \times 10^{-13} \]
- \[ 6.626 \times 10^{-34}/(9.109 \times 10^{-31} \times 3.00 \times 10^8)/(1/(1/v_2^2-1))^{0.5} = 8.80 \times 10^{-13} \]
- \[ 6.626 \times 10^{-34}/(9.109 \times 10^{-31} \times 2.998 \times 10^8)/(1/(1/v_2^2-1))^{0.5} = 8.81 \times 10^{-13} \]