2. Geometric Optics and introduction to interference (Ch. 34, 35 1-3)

Due: 11:59pm on Sunday, February 7, 2016

To understand how points are awarded, read the Grading Policy for this assignment.

Spherical Mirror 1

Description: This problem requires students to determine the spherical mirror that would create a prescribed image.

You wish to create an image that is 10 meters from an object. This image is to be inverted and half the height of the object. You wish to accomplish this using one spherical mirror.

Part A

What is the focal length $f$ of the mirror that would accomplish this?

Express your answer in meters, as a fraction or to three significant figures.

Hint 1. Find the focal length

There is a simple equation relating $s_{ob}$ (the object distance) and $s_{im}$ (the image distance) to $f$. Use this equation to obtain $f$ in terms of $s_{ob}$ and $s_{im}$.

Express your answer in terms of $s_{ob}$ and $s_{im}$.

ANSWER:

$$f = \frac{s_{ob} s_{im}}{s_{ob} + s_{im}}$$

If you are having trouble finding the values of $s_{ob}$ and $s_{im}$, Parts A.2 and A.3 should be helpful.

Hint 2. Find the magnification

The magnification $m$ is defined as the height of the image $y_{im}$ divided by the height of the object $y_{ob}$ . Find the magnification of the desired mirror.

Express your answer as a fraction.

ANSWER:

$$m = -0.500$$

Hint 3. Use the magnification to find $s_{ob}$ and $s_{im}$
A second definition of magnification is \( m = -\frac{s_{im}}{s_{ob}} \). Using the magnification you calculated in the Part A.2, find \( s_{ob} \) in terms of \( s_{im} \).

**Express your answer in terms of \( s_{im} \).**

**ANSWER:**

\[ s_{ob} = 2s_{im} \]

Now that you know that \( s_{ob} = 2s_{im} \), look again at the introduction to the problem. You should be able to find a second, simple equation relating \( s_{ob} \) and \( s_{im} \), based on the information in the introduction. Solve the two equations to obtain numerical values of \( s_{ob} \) and \( s_{im} \).

**ANSWER:**

\[ f = 6.67 \text{ m} \]

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**Part B**

Is this a concave or convex mirror?

**ANSWER:**

- [ ] concave  
- [ ] convex

Remember that concave mirrors have positive focal lengths, and convex mirrors have negative focal lengths. You calculated a positive focal length in Part A, so the mirror must be concave.

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**Part C**

What is the magnitude \( r \) of the radius of curvature of this mirror?

**Express your answer in meters, as a fraction or to three significant figures.**

**Hint 1. Relating focal length and radius of curvature**

For spherical mirrors, the focal length \( f \) is half of the radius of curvature \( r \) (i.e., \( f = r/2 \)).

**ANSWER:**

\[ r = 13.3 \text{ m} \]

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**Part D**

What type of image is created, real or virtual?

**Hint 1. Real versus virtual**

A real image is one that forms at a location where the light actually reaches. (Imagine a slide projector creating an image on a screen.) A virtual image is one that forms at a location that the light does not actually reach. (Imagine the image created when you look in a mirror hanging on a wall. No light actually makes it past the wall. There only appears to be a
ANSWER:

Real images have positive image distance, and virtual images have negative image distance. Since you calculated a positive image distance in Part A, the image must be real.

Part A

Find the focal length of the lens that produces the image described in the problem introduction using the thin lens equation.

Express your answer in centimeters, as a fraction or to three significant figures.
Part B

Considering the sign of $f$, is the lens converging or diverging?

ANSWER:

- converging
- diverging

Part C

What is the magnification $m$ of the lens?

Express your answer as a fraction or to three significant figures.

ANSWER:

$m = -1.25$

Part D

Think about the sign of $s'$ and the sign of $y'$, which you can find from the magnification equation, knowing that a physical object is always considered upright. Which of the following describes the nature and orientation of the image?

ANSWER:

- real and upright
- real and inverted
- virtual and upright
- virtual and inverted

Now consider a diverging lens with focal length $f = -15$ cm, producing an upright image that is $5/9$ as tall as the object.

Part E

Is the image real or virtual? Think about the magnification and how it relates to the sign of $s'$.

ANSWER:

- real
- virtual

Part F

What is the object distance? You will need to use the magnification equation to find a relationship between $s$ and $s'$. Then substitute into the thin lens equation to solve for $s$.

Answer in centimeters, as a fraction or to three significant figures.
Part G
What is the image distance?
Express your answer in centimeters, as a fraction or to three significant figures.
ANSWER:
\[ s' = -6.67 \text{ cm} \]
A lens placed at the origin with its axis pointing along the \( x \) axis produces a real inverted image at \( x = -24 \text{ cm} \) that is twice as tall as the object.

Part H
What is the image distance?
Express your answer in centimeters, as a fraction or to three significant figures.
ANSWER:
\[ s' = 24.0 \text{ cm} \]

Part I
What is the \( x \) coordinate of the object? Keep in mind that a real image and a real object should be on opposite sides of the lens.
Express your answer in centimeters, as a fraction or to three significant figures.
ANSWER:
\[ x = 12.0 \text{ cm} \]

Part J
Is the lens converging or diverging?
ANSWER:
- converging
- diverging
You can solve the lens equation for \( s' \) in terms of \( s \) and \( f \). If you do this and then substitute your result into the magnification equation, you will see that the only way to obtain an image of a real object that is larger than the object itself is with a converging lens.

Part K
Find the focal length of the lens.
Type setting math:
A Convex-Convex Lens

Description: Lensmaker's equation for a lens in air, then a lens in water.

A "biconvex" lens is one in which both surfaces of the lens bulge outwards. Suppose you had a biconvex lens with radii of curvature $|R_1| = 10 \text{ cm}$ and $|R_2| = 15 \text{ cm}$. The lens is made of glass with index of refraction $n_{\text{glass}} = 1.5$. We will employ the convention that $R_1$ refers to the radius of curvature of the surface through which light will enter the lens, and $R_2$ refers to the radius of curvature of the surface from which light will exit the lens.

Part A

Is this lens converging or diverging?

ANSWER:

- [ ] converging
- [x] diverging

Part B

What is the focal length $f$ of this lens in air (index of refraction for air is $n_{\text{air}} = 1$)?

Express your answer in centimeters to two significant figures or as a fraction.

**Hint 1. The signs of the radii of curvature**

The sign convention for the lensmaker's equation is that a radius of curvature is negative if the center of curvature is on the same side of the lens as the object, and is positive if the center of curvature is on the opposite side of the lens from the object.

**Hint 2. The lensmaker's equation**

The lensmaker's equation is \[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right), \] where $f$ is the focal length, $n$ the refractive index of the lens material, and $R_1$ and $R_2$ the radii of curvature.

ANSWER:

$\ f = 12 \text{ cm}$

Part C

What is the focal length of the lens if it is immersed in water ($n_{\text{water}} = 1.3$)?

Express your answer in centimeters, to two significant figures or as a fraction.

**Hint 1. Putting the lens in water**

You can account for a lens being in a medium other than air by replacing $n$ in the lensmaker's equation with $\frac{n_{\text{water}}}{n_{\text{medium}}}$. Notice that if you let $n_{\text{medium}} = 1$, which is the case for air, you recover the original equation.
Focusing with the Human Eye

Description: A man looks at three objects at different distances. Explain how he must adjust the lenses in his eyes to bring the various objects into focus.

Joe is hiking through the woods when he decides to stop and take in the view. He is particularly interested in three objects: a squirrel sitting on a rock next to him, a tree a few meters away, and a distant mountain. As Joe is taking in the view, he thinks back to what he learned in his physics class about how the human eye works.

Light enters the eye at the curved front surface of the cornea, passes through the lens, and then strikes the retina and fovea on the back of the eye. The cornea and lens together form a compound lens system. The large difference between the index of refraction of air and that of the aqueous humor behind the cornea is responsible for most of the bending of the light rays that enter the eye, but it is the lens that allows our eyes to focus. The ciliary muscles surrounding the lens can be expanded and contracted to change the curvature of the lens, which in turn changes the effective focal length of the cornea-lens system. This in turn changes the location of the image of any object in one’s field of view. Images formed on the fovea appear in focus. Images formed between the lens and the fovea appear blurry, as do images formed behind the fovea. Therefore, to focus on some object, you adjust your ciliary muscles until the image of that object is located on the fovea.

Part A

Joe first focuses his attention (and his eyes) on the tree. The focal length of the cornea-lens system in his eye must be _________ the distance between the front and back of his eye.

**Hint 1. How to approach the problem**

Draw a picture of the object (the tree), the lens, and the image it produces. Be sure to include the focal point of the lens. Where must the fovea be in your sketch if this object is in focus? Is the focal point between the lens and the fovea, on the fovea, or behind the fovea?

**ANSWER:**

- greater than
- less than
- equal to

Part B

Joe’s eyes are focused on the tree, so the squirrel and the mountain appear out of focus. This is because the image of the squirrel is formed _______ and the image of the mountain is formed _______.

**Typesetting math:**
**Hint 1. Image of the squirrel**

The squirrel is closer to the lens (the eye) than the tree. As long as Joe’s eyes stay fixed on the tree, their focal length does not change. Using the lens equation, determine whether the image of the squirrel is closer to the lens than the image of the tree or farther away.

**Hint 2. Image of the mountain**

The mountain is farther from the lens (the eye) than the tree. As long as Joe’s eyes stay fixed on the tree, their focal length does not change. Using the lens equation, determine whether the image of the mountain is closer to the lens than the image of the tree or farther away.

**ANSWER:**

- between the lens and fovea / between the lens and fovea
- between the lens and fovea / behind the fovea
- behind the fovea / between the lens and fovea
- behind the fovea / behind the fovea

**Part C**

Joe now shifts his focus from the tree to the squirrel. To do this, the ciliary muscles in his eyes must have _____ the curvature of the lens, resulting in a(n) _______ focal length for the cornea-lens system. Note that curvature is different from radius of curvature.

**ANSWER:**

- increased / increased
- increased / decreased
- decreased / increased
- decreased / decreased

**Part D**

Finally, Joe turns his attention to the mountain in the distance but finds that he cannot bring the mountain into focus. This is because he is nearsighted. But when Joe puts on his glasses, he can see the mountain clearly. To adjust for his nearsightedness, his glasses must contain _____ lenses.

**Hint 1. Focusing on distant objects**

The image of a distant object like the mountain always forms at (or very close to) the focal point of the fovea-lens system. When Joe is looking at the most distant object he can see clearly, where is the focal point?

**Hint 2. The role of corrective lenses**

Nearsightedness and farsightedness are both caused by the fact that the ciliary muscles cannot make the focal length of the lens arbitrarily large or small. The corrective lenses must make the image of the distant mountains form someplace that his eyes are naturally able to focus on.

**ANSWER:**

*Typesetting math:*
Constructive and Destructive Interference Conceptual Question

Description: Conceptual question on whether constructive or destructive interference occurs at various points between two wave sources.

Two sources of coherent radio waves broadcasting in phase are located as shown below. Each grid square is 0.5 m square, and the radio sources broadcast at $\lambda = 2.0$ m.

Part A

At Point A is the interference between the two sources constructive or destructive?

**Hint 1. Path-length difference**

Since the two sources emit radio waves in phase, the only possible phase difference between the waves at various points is due to the different distances the waves have traveled to reach those points. The difference in the distances traveled by the two waves from source to point of interest is termed the path-length difference.

If the path-length difference is an integer multiple of the wavelength of the waves, one wave will pass through an integer number of complete cycles more than the other wave, placing the two waves back in perfect synchronization, resulting in constructive interference. If the path-length difference is a half-integer multiple of the wavelength, one wave will be one-half of a cycle, or 180 degrees, out of phase, resulting in destructive interference.

**Hint 2. Find the path-length difference**

What is the distance from the left source to Point A, $d_{A,\text{left}}$? What is the distance from the right source to Point A, $d_{A,\text{right}}$?

Enter the distances in meters separated by a comma.

**ANSWER:**

$d_{A,\text{left}} \cdot d_{A,\text{right}} = 3, 3$ m.

The wave that leaves the source on the left and the wave that leave the source on the right travel equal distances to Point A.

**Typesetting math:**

$\sqrt{\frac{3}{3}}$
Part B
At Point B is the interference between the two sources constructive or destructive?

**Hint 1. Find the path-length difference**
What is the distance from the left source to Point B, $d_{B,\text{left}}$? What is the distance from the right source to Point B, $d_{B,\text{right}}$?

Enter the distances in meters separated by a comma.

**ANSWER:**

\[ d_{B,\text{left}}, d_{B,\text{right}} = 1.5, 4.5 \text{ m, m} \]

The path-length difference between the two waves is 3 m.

**ANSWER:**

- constructive
- destructive

Part C
At Point C is the interference between the two sources constructive or destructive?

**ANSWER:**

- constructive
- destructive

Part D
At Point D is the interference between the two sources constructive or destructive?

**ANSWER:**

- constructive
- destructive

Double Slit 1
**Description:** This problem explores double-slit interference maxima and minima.

Two lasers are shining on a double slit, with slit separation $d$. Laser 1 has a wavelength of $d/20$, whereas laser 2 has a wavelength of $d/15$. The lasers produce separate interference patterns on a screen a distance 5.80 m away from the slits.

**Typesetting math:**
Part A

Which laser has its first maximum closer to the central maximum?

**Hint 1. Path difference**

The first maximum comes when the path difference between the two slits is equal to one full wavelength. Think about which laser has a smaller wavelength, and recall that the distance from the central maximum is proportional to the path difference.

**ANSWER:**

- laser 1
- laser 2

Part B

What is the distance $\Delta y_{\text{max-max}}$ between the first maxima (on the same side of the central maximum) of the two patterns?

**Express your answer in meters.**

**Hint 1. Find the location of the first maximum for laser 1**

The first maximum corresponds to constructive interference, with $m = 1$ (since the central maximum corresponds to $m = 0$). Using the small-angle approximation, what is the distance $y_1$ of this maximum from the central maximum for laser 1?

**Express your answer in meters.**

**Hint 1. Angle to maxima**

The angle to the $m$th maximum is given by $d \sin(\theta) = m\lambda$, where $\theta$ is the angle, $d$ is the separation between the slits, and $\lambda$ is the wavelength of the light.

**Hint 2. Distance on screen**

For a screen that is far from the slits, as in this problem, the distance $y$ on the screen from the central maximum is $y = R \sin(\theta)$, where $\theta$ is the angle from the slits to the point on the screen and $R$ is the distance from the slits to the screen.

**ANSWER:**

$y_1 = \frac{L}{20} = 0.290 \text{ m}$

**Hint 2. Find the location of the first maximum for laser 2**

Now that you have found the first maximum for laser 1, what is the location $y_2$ of the first maximum for laser 2?

**Express your answer in meters.**

**Hint 1. Angle to maxima**

The angle to the $m$th maximum is given by $d \sin(\theta) = m\lambda$, where $\theta$ is the angle, $d$ is the separation between the slits, and $\lambda$ is the wavelength of the light.
**Hint 2. Distance on screen**

For a screen that is far from the slits, as in this problem, the distance $y$ on the screen from the central maximum is

$$y = R \sin(\theta),$$

where $\theta$ is the angle from the slits to the point on the screen and $R$ is the distance from the slits to the screen.

**Answer:**

$$y_2 = \frac{L}{15} = 0.387 \text{ m}$$

To calculate the distance between these two maxima, just calculate the difference of the two distances.

**Answer:**

$$\Delta y_{\text{max-max}} = \frac{L}{60} = 9.67 \times 10^{-2} \text{ m}$$

Also accepted: $\frac{L}{59.7} = 9.72 \times 10^{-2}$

**Part C**

What is the distance $\Delta y_{\text{max-min}}$ between the second maximum of laser 1 and the third minimum of laser 2, on the same side of the central maximum?

**Express your answer in meters.**

**Hint 1. Find the location of the second maximum**

If the central maximum corresponds to $m = 0$, then you should be able to figure out what the second maximum corresponds to. Using that, what is the distance $y_1$ to the second maximum of laser 1 from the central maximum?

**Express your answer in meters.**

**Hint 1. Angle to maxima**

The angle to the $m$th maximum is given by $d \sin(\theta) = m\lambda$, where $\theta$ is the angle, $d$ is the separation between the slits, and $\lambda$ is the wavelength of the light.

**Hint 2. Distance on screen**

For a screen that is far from the slits, as in this problem, the distance $y$ on the screen from the central maximum is

$$y = R \sin(\theta),$$

where $\theta$ is the angle from the slits to the point on the screen and $R$ is the distance from the slits to the screen.

**Answer:**

$$y_1 = \frac{L}{10} = 0.580 \text{ m}$$

Also accepted: $\frac{L}{9.95} = 0.583$
**Hint 2. Find the $m$ value of the third minimum**

The first minimum corresponds to $m = 0$ (since there is no central minimum). What, then, is the value of $m$ for the third minimum? Recall that $m$ is always an integer.

**Express your answer as a whole number.**

**ANSWER:**

\[ m_{\text{third minimum}} = 2 \]

**Hint 3. Find the location of the third minimum**

Given that $m = 2$, what is the location of the third minimum?

**Express your answer in meters.**

**Hint 1. Difference between the angle to a maximum and a minimum**

Once you have the value of $m$, the equation for the angle to a minimum is almost identical to the equation for the angle to a maximum. The only difference is that you use $m + 1/2$ in place of $m$ if you want to find the location of a minimum instead of a maximum.

**ANSWER:**

\[ y_{\text{third minimum}} = \frac{L}{6} = 0.967 \text{ m} \]

Also accepted: \[ \frac{L}{5.92} = 0.980 \]

\[ \Delta y_{\text{max-min}} = \frac{L}{15} = 0.387 \text{ m} \]

Also accepted: \[ \frac{L}{14.6} = 0.397 \]