Exercise 33.5

Description: A light beam travels at 1.94 * 10^8 m/s in quartz. The wavelength of the light in quartz is 355 nm. (a) What is the index of refraction of quartz at this wavelength? (b) If this same light travels through air, what is its wavelength there?

A light beam travels at 1.94 * 10^8 m/s in quartz. The wavelength of the light in quartz is 355 nm.

Part A

What is the index of refraction of quartz at this wavelength?

ANSWER:

\[ n = 1.55 \]

Part B

If this same light travels through air, what is its wavelength there?

ANSWER:

\[ \lambda = 5.49 \times 10^{-7} \text{ m} \]

Problem 33.48

Description: A 45 degree(s)-45 degree(s)-90 degree(s) prism is immersed in water. A ray of light is incident normally on one of its shorter faces. (a) What is the minimum index of refraction that the prism must have if this ray is to be totally reflected within...

A 45° — 45° — 90° prism is immersed in water. A ray of light is incident normally on one of its shorter faces.
Part A

What is the minimum index of refraction that the prism must have if this ray is to be totally reflected within the glass at the long face of the prism?

Express your answer using two significant figures.

ANSWER:

\( n = 1.9 \)
Note that the index of refraction of the glass doesn't figure at all in finding the angle of refraction in water. The mathematics reduces to:

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) = n_3 \sin(\theta_3) \]

or

\[ n_1 \sin(\theta_1) = n_3 \sin(\theta_3). \]

The final angle is the same as if the light ray were passing directly from the air into the water.

A message from your instructor...

For the problem "How Deep is the Goldfish?", it is very useful to look at Example 34.7 (page 1130 in Volume 13, or 1173 in Volume 12).

How Deep Is the Goldfish?

**Description:** Use the object-image relation and the law of refraction to determine the apparent distance to a goldfish underwater, as well as its reflection image at the bottom of the pool.

A tank whose bottom is a mirror is filled with water to a depth of 19.9 cm. A small fish floats motionless 6.40 cm under the surface of the water.

**Part A**

What is the apparent depth of the fish when viewed at normal incidence to the water?

Express your answer in centimeters. Use 1.33 for the index of refraction of water.

**Hint 1. Use the object-image relation**

Recall that the object-image relation for a spherical refracting surface is given by the equation

\[ \frac{n_a}{s_a} + \frac{n_b}{s_b} = \frac{n_b-n_a}{R}, \]

where \( s_a \) is the object distance in the material with index of refraction \( n_a \), \( s_b \) is the image distance in the material with index of refraction \( n_b \), and \( R \) is the radius of curvature of the surface.

**Hint 2. Radius of curvature of the surface**

Notice that the surface of the water in the fish bowl is essentially flat. As a result, the radius of curvature of this surface is infinite.

**ANSWER:**

\[ \theta_3 = \arcsin \left( \frac{\sin(\theta_a)}{n_{\text{water}}} \right) \frac{180}{\pi} = 23.5 \degree \]
Part B

What is the apparent depth of the reflection of the fish in the bottom of the tank when viewed at normal incidence?

Express your answer in centimeters. Use 1.33 for the index of refraction of water.

**Hint 1. Follow the path of light**

Keep in mind that the light has to bounce from the bottom of the fish, down to the mirror, and then back up and out of the water to the viewer. First think of how far down the fish would appear if the index of refraction were the same in the water as out. You can use the law of reflection for a plane mirror to determine where the fish would appear without the water. Then apply the correction for the real refraction due to the water.

**ANSWER:**

\[
\frac{2h_1 - h_2}{n} = 25.1 \text{ cm}
\]

---

**The Focal Length of a Lens**

**Description:** Short quantitative problem on magnification and focal length of converging and diverging lenses. The thin-lens equation is used. Based on Young/Geller Quantitative Analysis 24.4.

An object is located 30.0 cm from a certain lens. The lens forms a real image that is twice as high as the object.

**Part A**

What is the focal length of this lens?

**Hint 1. How to approach the problem**

To find the focal length of the lens you can use the thin-lens equation. To do that, you first need to calculate the object distance and the image distance. In particular, the image distance can be determined from the magnification of the lens, which can be computed since you know that the height of the image is twice the height of the object. Note that the lens forms a real image; therefore, it must be a converging lens and the image is inverted.

**Hint 2. Find the magnification**

What is the lateral magnification of a lens that produces a real image twice as high as the object?

**Hint 1. Magnification**

The lateral magnification \( m \) produced by a lens is the ratio of the image height \( y' \) to the object height \( y \). It is also equal to the negative ratio of the image distance \( s' \) to the object distance \( s \):
\[ m = \frac{y'}{y} = -\frac{s'}{s}. \]

The negative sign indicates that when \( s \) and \( s' \) are both positive, \( y \) and \( y' \) have opposite signs, and the image is inverted.

**ANSWER:**

- \( 2 \)
- \( -2 \)
- \( 1/2 \)
- \( -1/2 \)

Because the image is real, the lens must be a converging lens. Recall that converging lenses form real images that are *inverted*; therefore, their magnification is negative. It follows that both the object distance and the image distance are positive.

**Hint 3. Find the object distance**

The object is located 30.0 cm from the lens and the image produced by the lens is real. What is the object distance \( s \) measured from the lens?

Express your answer in centimeters

**Hint 1. Sign rule**

Since the lens forms a real image, it must be a converging lens and the object must be on the same side of the lens as the incoming rays. Therefore, the object distance is positive.

**ANSWER:**

\[ s = d = 30.0 \text{ cm} \]

**Hint 4. Find the image distance**

The object is located 30.0 cm from the lens and the image produced by the lens is real. What is the image distance \( s' \), measured from the lens, if the magnification of the lens is \(-2\)?

Express your answer in centimeters.

**Hint 1. Magnification**

The lateral magnification \( m \) produced by a lens is the ratio of the image height \( y' \) to the object height \( y \). It is also equal to the negative ratio of the image distance \( s' \) to the object distance \( s \). That is,

\[ m = \frac{y'}{y} = -\frac{s'}{s}. \]

The negative sign tells us that when \( s \) and \( s' \) are both positive, \( y \) and \( y' \) have opposite signs, and the image is inverted.
ANSWER:

\[ s' = 2d = 60.0 \text{ cm} \]

Now use the thin-lens equation to calculate the focal length of the lens.

**Hint 5. Thin-lens equation**

The thin-lens equation expresses a useful relation between the focal length \( f \) of a lens, the object distance \( s \), and the image distance \( s' \). It can be written as

\[
\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}
\]

ANSWER:

- 90.0 cm
- 10.0 cm
- 10.0 cm
- 5.00 cm
- 20.0 cm

**Part B**

Now replace the lens used in Part A with another lens. The new lens is a diverging lens whose focal points are at the same distance from the lens as the focal points of the first lens. If the object is 5.00 cm high, what is the height of the image formed by the new lens? The object is still located 30.0 cm from the lens.

**Hint 1. How to approach the problem**

To find the image height you need to know the magnification of the new lens, which can be calculated if you know the image distance and the object distance. To find the image distance you can use the thin-lens equation, but be sure to use the correct value for the focal length. Recall that the focal length of a diverging lens is a negative quantity.

**Hint 2. Find the focal length of the new lens**

The lens used in Part A has been replaced with a diverging lens whose focal points are at the same distance from the lens as the focal points of the first lens. What is the focal length of the diverging lens?

**Express your answer in centimeters.**

**Hint 1. Focal length of a diverging lens**

The focal length of a diverging lens is a negative quantity. Its focal points are virtual points, and they are reversed relative to the focal points of a converging lens.

ANSWER:
**Hint 3. Find the image distance**

After you have calculated the focal length of the new lens, find the new image distance \( s' \).

**Express your answer in centimeters.**

**Hint 1. Thin-lens equation**

The thin-lens equation expresses a very useful relation between the focal length \( f \) of a lens, the object distance \( s \), and the image distance \( s' \). It can be written as

\[
\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}.
\]

**ANSWER:**

\[
s' = \frac{-2d}{5} = -12.0 \text{ cm}
\]

**Hint 4. Find the magnification**

Once you have computed the new image distance, find the magnification \( m \) produced by the new lens.

**Express your answer numerically.**

**Hint 1. Magnification**

The lateral magnification \( m \) produced by a lens is the ratio of the image height \( y' \) to the object height \( y \). It is also equal to the negative ratio of the image distance \( s' \) to the object distance \( s \). That is,

\[
m = \frac{y'}{y} = -\frac{s'}{s}.
\]

The negative sign tells us that when \( s \) and \( s' \) are both positive, \( y \) and \( y' \) have opposite signs, and the image is inverted.

**ANSWER:**

\[
m = 0.400
\]

Now that you know the magnification, you can use it to compute the image height. To do that, recall that the magnification is defined as the ratio of the image height to the object height.

**ANSWER:**

\[
\]
Where Is the Image?

**Description:** Ray tracing to find image location and size for a convex lens, converging and diverging cases. (vector applet)

**Learning Goal:**
To understand how to use ray tracing to determine image location and size for a convex lens.

Ray tracing is one way to determine the location and size of an image formed by a lens. Ray tracing allows you to follow the path of a few principal rays originating from a point on the imaged object. The thin lens approximation simplifies ray tracing by ignoring details of light propagation inside the lens.

A converging lens focuses a set of light rays entering the lens on one side parallel to its axis to a single focal point on the opposite side of the lens. If the principal rays emitted from a single point on an object all intersect at a single point after passing through a converging lens, then this intersection point helps in determining the location and size of a real image of the object. The most useful principal rays for converging lenses are the following:

1. The $P$ ray is directed parallel to the axis of the lens. It emerges from the lens directed toward the focal point on the side of the lens opposite the object.
2. The $M$ ray is directed toward the midpoint of the lens. It emerges from the lens unchanged in direction.
3. The $F$ ray is directed toward the focal point on the same side of the lens as the object. It emerges from the lens directed parallel to the lens axis.

**Part A**

Trace the $P$ ray and $M$ ray from the tip of the object for the converging lens shown (use the label $P_1$ for the segment of the $P$ ray on the same side as the object and $P_2$ for the segment on the opposite side). The points labeled $F$ are the focal points of the lens. Be sure to extend the $M$ ray enough to intersect the $P$ ray.

**Draw the vectors for the incident rays starting from the tip of the object. The location and orientation of the vectors will be graded.**

**Hint 1. Path of the $P$ ray**

For a converging lens, the $P$ ray is initially parallel to the axis of the lens. It emerges from the lens on the opposite side directed toward the focal point.

**Hint 2. Path of the $M$ ray**

The $M$ ray is initially directed toward the midpoint of the lens. It emerges from the lens without changing direction.

**ANSWER:**
Part B

If the focal length (the distance from the lens to either focal point \( F \)) of the lens is \( f \), which of the following is true of the horizontal distance \( d_i \) from the lens to the image?

**Hint 1. Location of the image formed by the lens**

Recall that the location of the image found by ray tracing is determined by the intersection point of two or more principal rays.

**ANSWER:**

- \( d_i < f \)
- \( f < d_i < 2f \)
- \( d_i > 2f \)

Part C

Now add the \( F \) ray to your diagram (use the label \( F_1 \) for the segment of the \( F \) ray on the same side as the object and \( F_2 \) for the segment on the opposite side).

Draw the vectors starting from the tip of the object. The location and orientation of the vectors will be graded.

**Hint 1. Path of the \( F \) ray**
For a converging lens, the $F$ ray, or focal-point ray, is a ray that passes through the focal point located on the same side of the lens as the object and emerges parallel to the axis of the lens.

ANSWER:

As you can see, all three principal rays pass through the same intersection point. In fact, all rays from a single point on the object will pass through a single point on the opposite side of a converging lens. Therefore you can use the intersection point of any two rays to find the location of the image; the principal rays are usually the easiest ones to use.

Part D

What happens to the image formed by the same lens if the distance between the object and the focal point is reduced? Assume that the object remains to the left of the focal point.

**Hint 1. How to approach the problem**

Simply sketch the ray diagram shown in Part B or D and find the image as you did in Part C. Then, on the same diagram, draw the same object at a new location closer to the focal point $F$. Then trace the same two principal rays that you used to find the image in the first place, and determine the location of the new image. Now compare your results. Notice that the $P$ ray is unchanged as you move the object along the axis of the lens.

ANSWER:
The situation described so far is typical of convex lenses. Whenever the object is farther from the lens than the focal point, the lens creates a real image that is inverted. If the distance between the object and the focal point is reduced, the size of the image is increased. This is basically what happens in a camera. When you zoom in, you are actually moving the lens closer to the object, reducing the distance between the object and the focal point of the lens. As a result, the size of the image on the film increases.

Part E

Now consider the case in which the object is between the lens and the focal point. Trace the $P$ ray (use the label $P_1$ for the segment of the $P$ ray on the same side as the object and $P_2$ for the segment on the opposite side) and the $M$ ray.

Draw the vectors for the incident rays starting from the tip of the object. The location and orientation of the vectors will be graded.

ANSWER:
In this case the $P$ and $M$ rays diverge. The lens now forms a virtual image that can be found by extending the diverging rays backward, as shown in the figure.

A convex lens can act as both a converging lens and a diverging lens. It forms a real image when the object is located at a distance from the lens greater than the distance of the focal point, while it forms a virtual image when the object is between the focal point and the lens.

The Magnification Produced by a Lens

**Description:** Short conceptual problem on lateral magnification produced by a lens. Based on Young/Geller Conceptual Analysis 24.3.

**Part A**

What can one say about the image produced by a thin lens that produces a positive magnification?

**Hint 1. How to approach the problem**

When a lens produces an inverted image, the image lies below the lens axis and its height is taken to be negative. In such a case, the ratio of the image height to the object height is negative. Relate this to the magnification produced by the lens and the image location.

**Hint 2. Lateral magnification**

The lateral magnification $m$ produced by a lens is the ratio of the image height $y'$ to the object height $y$:

$$m = \frac{y'}{y}.$$  

This means that when the magnification produced by a certain lens is negative, the image height and the object height have opposite signs.

**Hint 3. Find another expression for the magnification produced by a thin lens**

For a lens that produces an image at $s'$ of an object located at $s$, which of the following expressions gives the correct magnification produced by the lens?

**ANSWER:**

Typesetting math:
Thus, for a lens that produces a positive magnification of a real object, is the image distance positive or negative?

**Hint 4. Sign rule for image distances**

When the image is on the same side of the lens as the outgoing light, the image distance is positive; otherwise, it is negative. Also, remember that when the outgoing rays pass through an image point, the image is called a *real* image. However, when the outgoing rays do not pass through an image point and instead appear to diverge as if they had come from an image point, the image is a *virtual* image.

**ANSWER:**

- It is real and inverted.
- It is real and erect.
- It is virtual and inverted.
- It is virtual and erect.

Note that the magnification produced by a lens is not a constant property of the lens. In fact, it can change depending on the location of the object. That is, the same lens can produce a positive magnification for a certain object distance from the lens, and a negative magnification for a different object distance.

**Part B**

If the diameter of a lens is reduced, what happens to the magnification produced by the lens?

**Hint 1. How to approach the problem**

The magnification produced by a lens depends on the image formed by the lens, which is determined exclusively by the position of the object relative to the lens and the optical properties of the lens.

**ANSWER:**

- It increases.
- It decreases.
- It is unchanged.
Lens Producing an Image Conceptual Question

Description: Simple conceptual question about deducing what kind of lens is creating a certain type of image.

A lens produces a real image of a real object.

Part A

Is the image inverted or upright?

**Hint 1. Orientation**

A real image is on the opposite side of the lens from the real object. Considering the path of the M ray (the ray passing through the midpoint of the lens) should allow you to determine the orientation of the image.

**ANSWER:**

- inverted
- upright
- cannot be determined

Part B

Is the lens diverging or converging?

**ANSWER:**

- diverging
- converging
- cannot be determined

Part C

Is the image enlarged or reduced in size?

**Hint 1. Determine the object's location**

Is the object within the focal length or beyond the focal length of the converging lens?

**ANSWER:**

- within
- beyond

**Hint 2. Principal rays for possible object location**
The object must be beyond the focal length. The principal ray diagram below shows the object at twice the focal length. If the object is at a different location beyond the focal length, the image size can be determined by a similar diagram.

![Principal ray diagram](image.png)

**Hint 3. Find the change in image size if the object is moved closer**

If the object is at twice the focal length, the image is exactly the same size as the object. As the object is moved closer to the lens, does the image become larger or smaller than the object?

**ANSWER:**

- larger
- smaller

**Hint 4. Find the change in image size if the object is moved farther away**

If the object is at twice the focal length, the image is exactly the same size as the object. If the object is moved farther from the lens, does the image become larger or smaller than the object?

**ANSWER:**

- larger
- smaller

**ANSWER:**

- enlarged
- reduced
- cannot be determined

**Part D**

If two convex lenses identical in size and shape are manufactured from glass with two different indices of refraction,
would the focal length of the lens with the greater index of refraction (lens 1) be larger or smaller than that of the other lens (lens 2)?

**Hint 1. Index of refraction**
The larger the difference between the refractive index of the lens and the refractive index of the surrounding air, the more the light will be bent by the action of the lens.

**Hint 2. Focal length**
The larger the focal length, the less effect the lens has on the path of the light.

ANSWER:
- larger
- smaller
- cannot be determined

---

**Part E**

If lens 1 from Part D were placed in exactly the same location as lens 2, would the image produced by lens 1 be larger or smaller than the image produced by lens 2?

**Hint 1. How to approach the problem**
First consider whether the object is within or beyond the focal point of lens 2. Will this change when the focal length is shortened? Now imagine you are measuring the object distance in units of focal lengths. The actual distance stays the same, but does the distance in units of focal lengths increase or decrease when the focal length is shortened? Recall what happens to the size of an image as the object is moved away from or toward the focal point.

**Hint 2. Location of the object**
The object must be beyond the focal length of a lens in order to form a real image. This means that the object is beyond the focal length of lens 2. In Part D, you found the focal length of lens 2 to be larger than the focal length of lens 1. Is the object within or beyond the focal length of lens 1?

**Hint 3. Image size when object is located at the focal point**
If an object is located exactly at the focal point, the image is "infinitely" large. As the object moves away from the focal point, in either direction, the image size decreases.

**Hint 4. Ratio of distance to focal length**
One does not need to know the exact numerical values of the object distance and focal length to determine the magnification of the image. It is the ratio of the object distance to the focal length that is decisive. For objects beyond the focal length, as this ratio increases, the image size decreases.

ANSWER:
Which of the following cannot be determined?

- larger
- smaller
- cannot be determined