Physics 228

Thin Film Interference
Interferometry
Diffraction
Interference in Thin Films

It looks like if the path lengths are different by \( m \) (\( m \) integer) wavelengths, we have constructive interference, but if the difference is \( (m + \frac{1}{2}) \) wavelengths, there is destructive interference.

Note: Recall that the wavelength of light in a medium is \( \lambda_{\text{medium}} = \frac{\lambda_{\text{vacuum}}}{n} \), and you need to use \( \lambda_{\text{medium}} \) when determining the path length difference. We will just call it \( \lambda \).
Interference in Thin Films

(a) Interference between rays reflected from the two surfaces of a thin film

Light reflected from the upper and lower surfaces of the film comes together in the eye at $P$ and undergoes interference.

Some colors interfere constructively and others destructively, creating the color bands we see.

(b) The rainbow fringes of an oil slick on water

BUT:

We need to consider possible phase changes when the light refracts into a medium or reflects from a surface!
Phase Changes

The transmitted wave has the same sign as the incident wave. The reflected wave has either the same sign, or “flips” to the opposite sign, depending on whether reflection occurs from a less dense or a denser medium, respectively. For sine waves: Sign change = 180 degree phase shift!

Simulation
The situation with light is similar to the situation with the rope. The transmitted / refracted wave has the same phase as the incident wave. The reflected wave has the same phase going from a higher $n$ to lower $n$ medium, but the opposite phase when going from a lower $n$ to higher $n$ medium.
Why the Phase Changes?

This is a topic for Physics 305: Modern Optics, or an E&M course.

The general idea is the following:

• According to Maxwell’s equations, the parallel component of the total electric field on the incident side (incident + reflected fields) must be the same as the total field (refracted) on the other side.
• The same is true for the perpendicular component of the magnetic field.
• From these “boundary conditions”, the amplitudes and phases of the reflected and refracted waves follow. (“Fresnel equations”)
• Note on the side: For total internal reflection, the phase change is non-trivial and depends on angle of incidence.
Reflected Field for Normal Incidence

For light normally incident from medium $a$ to medium $b$:

$$E_r = \frac{n_a - n_b}{n_a + n_b} E_i$$

If the light is incident from the larger $n$ medium: $E_r$ is the same sign as $E_i$ (no phase change).

If the light is incident from the smaller $n$ medium: $E_r$ is the opposite sign as $E_i$ (phase change).

The more similar the indices are, the smaller the numerator and the less light gets reflected. When the $n$'s are the same no light is reflected (remember the invisible beaker?)
Returning to the Oil Film

Oil has an index of refraction similar to glass, \( n \approx 1.50 \).
Water has \( n \approx 1.33 \).
Air has \( n = 1 \).

- There is a 180° phase shift when the light reflects from the air-oil boundary.
- There is no phase shift when light reflects from the oil-water boundary.
- For normal incidence, because of the added phase shift, we get destructive interference when \( 2t = m \lambda_{\text{oil}} \), and constructive interference when \( 2t = (m + \frac{1}{2}) \lambda_{\text{oil}} \).
A Thin Air Film Between Glass

- Light reflects from the glass-air interface, with no phase change.
- Second reflection from the air-glass interface below, with a phase change.
- If the angle of incidence is small we get destructive interference when \(2t = m\lambda\), and constructive interference when \(2t = (m + \frac{1}{2})\lambda\).
- The phase change is at the bottom rather than the top surface, but the result is the same as for oil on water, or soap film in air.
Anti-Reflection Coatings

• An important application is the design of anti-reflection coatings for optical elements.

• Consider a thin film of thickness $\lambda/4$, and an index of refraction $1 < n_{\text{coating}} < n_{\text{glass}}$.

• When light reflects from the top vs bottom of the coating, there is a $\lambda/2$ path length difference.

• Thus there is destructive interference for the reflected wave.

• $n_{\text{coating}}$ may be chosen such that all the light is transmitted.

• Since the wavelengths of visible light vary by nearly a factor of 2 this does not work equally well for all wavelengths at the same time.
Interferometers

- Interferometers measure changes in optical path length to high precision.
- The type shown here was used by Michelson & Morley in 1887 to show that the speed of light does not depend on whether the light path is along the earth’s motion around the sun, or perpendicular to it.
- The highest precision Michelson-Morley interferometer is LIGO, used to search for ripples in space-time (gravitational waves). It’s “legs” are 4 km long.
Are Shadows Infinitely Sharp?

Geometric optics predicts that this situation should produce a sharp boundary between illumination and solid shadow.

That’s NOT what really happens!

Point source

Area of illumination

Geometric shadow

Screen

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Are Shadows Infinitely Sharp?  

No. 

Many examples:  

- PhET simulation, physical demo

With a thin slit, you can see the light expand to both sides and form light and dark bands.
Why Does Light “Spill” Past the Geometrical Edge?

(a) A slit as a source of wavelets

Huygen’s Principle

Each point on the wave front can be viewed as a source for spherical waves propagating out from that point. The wave front is the envelope of all the secondary waves.

The sum of all these wavefronts will bend around corners.
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Diffraction = interference with many sources, not just a few.
Single Slit Diffraction

Idea: Mentally divide the slit into pairs of points, separated by $a/2$. Require the paths through each pair of points to interfere destructively. This is the case of the path length difference is $\lambda/2$.

For the two strips shown, the path difference to $P$ is $(a/2) \sin \theta$. When $(a/2) \sin \theta = 1/2$, the light cancels at $P$. This is true for the whole slit, so $P$ represents a dark fringe.

Condition for this destructive interference:

$$\lambda/2 = (a/2)\sin \theta$$

$$\Rightarrow \sin \theta = \lambda/a$$
Single Slit Diffraction

Are there other minima?

- Divide the slit into $2m$ sections ($m > 0$).
- Require section $i$ to interfere destructively with section $i+1$.
- Condition for destructive interference:
  \[ \frac{\lambda}{2} = \left(\frac{a}{2m}\right) \sin \theta \]
- Get minima for $\sin \theta = m\lambda/a$, with $m = \pm 1, \pm 2, \ldots$
- This argument does not work for $m = 0$!
At $m = 0$, there is a very bright central peak surrounded by much dimmer side peaks.

In the central peak ($m = 0$), all of Huygen’s wavelets are exactly in phase.
A single slit diffraction pattern with 500 nm light has a series of dark bands separated by 1 cm at a distance of 5 m from the slit. What is the width of the slit?

\[ \sin \theta = \frac{m \lambda}{a} \quad \text{or} \quad y = \frac{m R \lambda}{a} \]

Solve for \( a \):

\[ a = \frac{R \lambda}{y} \quad \text{(set} \ m = 1) \]

\[
\begin{align*}
a &= \frac{(5)(500 \times 10^{-9})}{0.01} \ m = 0.25 \ mm.
\end{align*}
\]