Lecture 24

Nuclear Physics
Radioactivity
Atomic Nuclei

The nucleus the center of the atom is positively charged and contains almost all the atom’s mass.

The size of the nucleus is about four to five orders of magnitude smaller than the atom. A typical nuclear radius is several femtometers (fm = 10^{-15} m).

Nuclei are made up of nucleons (protons and neutrons, each with similar mass). Protons have a charge +e (electron charge: -e), neutrons have zero charge.

Consider an atom of charge 3 (Li). Its nucleus has 3 protons, which we write as Z = 3.

How many neutrons are there? It turns out that there are two stable “isotopes” of Li, one with 3 neutrons, and one with 4 neutrons. These are called Li-6 and Li-7, respectively.

Most atoms have several stable isotopes (and several more that are not stable or radioactive).
Coulomb and Nuclear Forces

Since the protons repel each other, they must be held together by a force that is much stronger than the Coulomb repulsion. For this reason, the attractive force holding nuclei together is called the “strong” nuclear force.

The neutrons are subject to the same “strong” force potential as the protons, but only the protons experience the Coulomb force. As a result, the protons’ energy levels are higher in energy and tend to decay to lower-energy neutrons. This leads to stable nuclei that have more neutrons than protons.
Stability of Nuclei

Here we plot the lifetime of the nuclei (color, black is stable) in a chart of neutron number $N$ (y-axis) vs. proton number $Z$ (x-axis). There is a narrow “valley of stability” around an optimal neutron-to-proton ratio.
The volume of atomic nuclei is approximately proportional to the number of nucleons (think of a liquid droplet of nucleons).

Thus, the mass density of all nuclei is quite similar, \( \rho \approx 2.3 \times 10^{17} \text{ kg/m}^3 \) (compare with \( 10^3 \text{ kg/m}^3 \) for water!)

As a consequence, the nuclear radii are a simple function of the number of nucleons \( A \): \( r \approx R_0 A^{1/3} \) with \( R_0 = 1.2 \text{ fm} \).
Nuclear Masses

Nuclear masses are measured in a convenient unit: The atomic mass unit 1 u = 1.66053892 \times 10^{-27} \text{ kg}.

The a.m.u is defined as 1/12 of the mass of the $^{12}\text{C}$ atom.

The mass of the proton is 1.0073 u.

The mass of the neutron is 1.0087 u.

The mass of the electron is 0.00055 u.

The $^{12}\text{C}$ atom consists of 6 electrons, 6 protons, and 6 neutrons, thus one would expect it to have a total mass of 12.099 u. Yet it is only 12.000 u. What is going on?

The nucleus consists of bound nucleons, not free nucleons. Binding in an attractive potential lowers the energy by an amount called the “binding energy”. But remember that mass is energy, and energy is mass, measured in different units (via $E = mc^2$). Thus the mass of the nucleus will be lower than the mass of the free nucleons by this “binding energy”, converted to mass units. (Note that the positive binding energy is a negative contribution to the total energy/mass!)
The plot shows the total binding energy divided by the number of nucleons. The binding energy per nucleon exhibits maximum at $A = 62$.

Thus, if two light nuclei fuse into a heavier nucleus, energy is released (nuclear fusion in the sun).

If a heavy nucleus splits into two light ones, energy is released (fission, nuclear power plant).

The curve reaches a peak of about 8.8 MeV/nucleon at $A = 62$, corresponding to the element nickel. The spike at $A = 4$ shows the unusual stability of the $^4_2\text{He}$ structure.
Natural Radioactivity

• **γ decay**: an excited state of a nucleus decays to a lower lying state, emitting a photon (γ-ray).

• **α decay**: an α-particle (consisting of 2 protons and 2 neutrons) is emitted.

• **β decay**: In a neutron-rich nucleus, a neutron is converted into a proton, emitting an electron (and another particle). A similar process may occur in proton-rich nuclei: A proton is converted into a neutron, emitting a positron (the positively charged anti-particle of the electron).

• **electron capture**: a proton is converted into a neutron by capturing an electron from an s-orbital. This is the only way for protons to convert to neutrons if there is insufficient energy to create a positron.
α Decay

Many heavy nuclei are energetically unstable for breakup into an α particle - a $^4$He nucleus - and a nucleus with $A-4Z-2$. The (rest) mass of the parent nucleus is larger than the combined (rest) masses of the two products. What about the relativistic masses?

A useful way to think of this is as an α-particle bound into a potential well. The α moves around inside the potential, constantly reflecting off the wall. But its wave function has an exponential tail under the potential barrier, resulting in some probability of tunneling through the potential barrier.
β Decay

• Free protons are stable but free neutrons have a lifetime of about 15 minutes.

• The neutron’s decay is: \( n \rightarrow p + e^- + \bar{\nu}_e \)

• For historical reasons, in radioactive decay the electron is also referred to as a \( \beta^- \) particle, and the positron as a \( \beta^+ \) particle.

• The energy given off in this reaction is 0.782 MeV. It corresponds to the (rest) mass difference between the neutron and the products.

• This process may also occur inside a neutron-rich nucleus, with a different lifetime and a different amount of energy released. (In a neutron-rich nucleus, the Fermi energy for neutrons is higher than that for protons.)

• The corresponding decay of protons, \( p \rightarrow n + e^+ + \nu_e \), is not energetically allowed for free protons, but it is allowed for protons in proton-rich nuclei. (In a proton-rich nucleus, the Fermi energy for protons is higher than that for neutrons.)
Shown above is a “cut” through the $Z$ vs. $N$ nuclear energy landscape, for constant $A = Z + N$. The cut is perpendicular to the “valley of stability”.

Nuclei on the neutron-rich side of the valley approach the valley bottom by decaying in $\beta^-$ processes, while proton-rich nuclei decay toward the bottom via $\beta^+$ processes.

A related process is electron capture. Here an atomic electron enters the nucleus and the process $e + p \rightarrow n + \nu_e$ occurs in the nucleus, so $\beta^- + {}^A Z \rightarrow {}^{A-1}Z + \nu_e$.

In neutron stars, this process occurs to reduce the number of electrons and protons while increasing the number of neutrons.
Nuclear Excited States

It is possible to excite a single nucleon into a higher “orbit” (shell model), while leaving the state other nucleons unchanged. This is called a single-particle excitation.

Thinking of nuclei as liquid drops, you can also imagine the entire drop to vibrate and rotate (Just like molecules!). Since all nucleons contribute to these motions, these are called “collective” excitations.

In many nuclear spectra, there are in fact vibrational excited states of energy \((n+1/2)\hbar\omega\), and rotational excited states of energy \(l(l+1)\hbar^2/2I\).

Some excited states are not as easy to classify. They are intermediate between single-particle and collective excitations, and are difficult to visualize. This is a manifestation of the difficult problem of “correlation” in quantum mechanics.
When nuclei decay they can decay to the ground state of the final nucleus, or to an excited state.

The excited states then decay by emitting photons (γ rays), to lower lying states or directly to the ground state.

Thus we end up in a different energy state of the same nucleus.
The number of nuclei that decay each second is obviously proportional to the number of available nuclei $N$:

$$\frac{dN}{dt} = -\lambda N,$$

where $\lambda$ is the decay rate constant.

The solution of this equation is

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau},$$

where the lifetime $\tau = 1/\lambda$.

Another useful measure is the half life, $t_{1/2} = \ln(2)\tau = 0.69\tau$.

The unit for decays is $1$ Becquerel = $1$ decay/s, but another common unit is the Curie, $1$ Ci = $3.7 \times 10^{10}$ Bq.