Physics 228: Lecture 17

Structure of the Atom
Bohr Model
Spontaneous and Stimulated Emission
Lasers

Please join Canvas conference while listening to lecture during regular lecture period
8:55 - 9:50 AM
Towards the Structure of the Atom

In 1897, J. J. Thomson had discovered the electron and measured its charge/mass ratio.

By 1909, R Millikan measured the electron charge by sticking electrons to much larger oil drops, and observing their motion under the forces of gravity and an electric field.

The resulting electron mass was much smaller than the mass of atoms. Thus, almost all the mass of the neutral atom is associated with its positive charge component, not with the negatively charged electron.
**Rutherford Backscattering**

In 1911, E. Rutherford led experiments scattering alpha particles ($^4$He nuclei) from gold atoms. Most of the α particles went straight through the gold foil, while a very small number were deflected at large angles:

1. Alpha particles are emitted by a radioactive element such as radium.
2. Small holes in a pair of lead screens create a narrow beam of alpha particles.
3. Alpha particles strike foil and are scattered by gold atoms.
4. A scattered alpha particle produces a flash of light when it hits a scintillation screen, showing the direction in which it was scattered.
Rutherford concluded that the positive charge (and most of the mass) of the atom is concentrated in a very compact region, the atomic “nucleus”.
Planetary Model of Atom

- positively charged, massive nucleus
- negatively charged, light electrons
- electrostatic attraction
- Together, these facts suggest electrons "orbiting" around the nucleus, like planets around the sun.
- Problem: The orbiting electrons would radiate light, carrying away energy
- Electrons would spiral into nucleus, atom would collapse.

According to classical physics:
- An orbiting electron is accelerating, so it should radiate electromagnetic waves.
- The waves would carry away energy, so the electron should lose energy and spiral inward.
- The electron’s angular speed would increase as its orbit shrank, so the frequency of the radiated waves should increase.

Thus, classical physics says that atoms should collapse within a fraction of a second and should emit light with a continuous spectrum as they do so.

In fact:
- Atoms are stable.
- They emit light only when excited, and only at specific frequencies (as a line spectrum).
Bohr Model of Hydrogen Atom

Since electrons behave as waves, Niels Bohr reasoned that in the atom they might form standing waves, or resonances, just like standing waves on a string.

![Standing Waves Diagram]
Bohr proposed that only those electron orbits are allowed where an integer number of wavelengths fit into a circular orbit:

Bohr Model: Allowed Waves on a RING!
Bohr’s hypothesis explains the stability of atoms, as well as the line spectra:

Electrons absorb or emit particular energy photons to transition between two orbits, but once they are in the lowest energy orbit they cannot emit any more energy.

Consider: $2\pi r_n = n\lambda_n$ where $n$ is the "principal quantum number".

From de Broglie, $p = h/\lambda$, we obtain $2\pi r_n = n\lambda_n = nh/p_n$. This means that the angular momentum $L_n = r_n p_n = nh/2\pi = n\hbar$.

An integral number of wavelengths in each orbit leads to an orbital angular momentum that is an integer multiple of $\hbar$. Angular momentum is "quantized".
Bohr Atom Radius

- Consider an electron in circular orbit about a proton of radius $r_n$. The circumference is $2\pi r_n$.
- The electrostatic attraction force between the electron and proton is $F = \frac{e^2}{4\pi \varepsilon_0 r_n^2}$.
- In a circular orbit, this force must be equal to the mass times the centripetal acceleration $m v_n^2 / r_n$.
- Multiply both sides of the force equation by $m r_n^3$ to obtain:
  $$m e^2 r_n / 4\pi \varepsilon_0 = m^2 v_n^2 r_n^2 = L_n^2.$$  
- Now we use the quantization of angular momentum: $L_n = m_e v_n r_n = n\hbar$.
- Solve for $r_n$: $r_n = \frac{n^2 \hbar^2 \varepsilon_0}{\pi m e^2} \equiv n^2 a_0$.
- The "Bohr radius" is $a_0 = 0.053$ nm = 53 pm.
Bohr Atom Energy Levels

- Solving for the potential energy in the n’th orbit gives:
  \[ U_n = -\frac{e^2}{4\pi\varepsilon_0 r_n} = -\frac{m_e e^4}{4\varepsilon_0^2 n^2 \hbar^2}. \]

- The kinetic energy is
  \[ K_n = \frac{p^2}{2m} = (n \frac{\hbar}{r_n})^2 / 2m = \frac{m_e e^4}{8\varepsilon_0^2 n^2 \hbar^2}. \]

- We see that \( K_n = -\frac{1}{2} U_n \) so the total energy of the hydrogen atom is
  \[ E_n = K_n + U_n = -\frac{m_e e^4}{8\varepsilon_0^2 n^2 \hbar^2}. \]

- Plugging in numbers for the constants, we obtain
  \[ E_n = -13.6 \text{ eV} / n^2. \]
Bohr Model Transition Energies

(a) “Permitted” orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series.
Rydberg Formula for Hydrogen

With $E_n = -13.6 \text{ eV} / n^2$, the Bohr model of the hydrogen atom predicts that a transition from an (upper) level “$m$” to (lower) level “$n$” will result in emission of a photon of energy

$$E = 13.6 \text{ eV} \left(\frac{1}{n^2} - \frac{1}{m^2}\right).$$

Despite the simplicity of Bohr’s model, this is exactly (well, almost...) what is observed:

Light emitted by hydrogen gas discharge: Balmer Series
Reduced Mass

• Our result $E_n = -m_e e^4/8\varepsilon_0^2 n^2 h^2$ is not quite right. Why?
• In our derivation we assumed that the nucleus is at rest, corresponding to infinite mass. However, since the mass of the proton is large compared with the electron, but not infinite, the proton and electron each orbit their common center of mass.
• The correction is to replace the electron mass in the Rydberg formula with a reduced mass:

$$1/m_r = 1/m_e + 1/m_p$$

$$m_r = 0.99946 \, m_e.$$  

• It is clear that this is a small effect. Yet, the spectral lines of the “heavy” isotope of hydrogen deuterium (one proton and one neutron in the nucleus) are shifted by 0.027% compared to the lines of hydrogen. This is how deuterium was discovered.
Bohr Model Takeaway

The Bohr atom is based on E&M plus the assumption that circular orbits have an integral number of wavelengths. It leads to:

Quantized radii
Quantized angular momenta
Quantized energies

The energy levels are predicted correctly, and the concept of angular momentum quantization is also correct.

However, the model predicts that the n’th energy level always has an angular momentum of n\hbar, which is not confirmed experimentally.

Thus, the Bohr model allows us to start understanding quantization, but it is not really right. Full understanding requires the theory of quantum mechanics.
The probability of a photon being absorbed must be proportional to the number of photons in the initial state (in a given mode). Let’s call this number \( n+1 \). The final state then contains \( n \) photons.

For the time-reversed process (photon emission), the initial state contains \( n \) photons, and the final state contains \( n+1 \) photons.

The rate of the time-reversed process should equal that of the forward process (“detailed balance”).

Detailed balance dictates that the probability of photon emission is thus also proportional to \( n+1 \).
Stimulated and Spontaneous Emission

- Photon emission rate: \( P = \text{const.} \times (n+1) \)
- We see that the photon emission rate consists of two terms: One proportional to the number of photons in the initial state, the other independent of that number.
- We call the first term “stimulated emission”, as it may be considered stimulated by the initial state photons.
- The second term is called “spontaneous emission” since it occurs even if there are no photons present initially.

\[ \text{(here } n = 1 \text{)} \]
Lasers

- **LASER =** Light Amplification by Stimulated Emission of Radiation.

- Stimulated emission rate is proportional to the number of photons already present: Number of photons grows exponentially, like an avalanche:

- To get a laser to work, we need more atoms in a higher-energy state than in a lower-energy state ("population inversion").

- This cannot happen in thermal equilibrium.

- How do we produce population inversion?
Consider a system with 4 electronic levels.
We first excite the system ("pump") by passing current or light through it.

Consider one of the excited states having a very long lifetime compared to the typical excitation lifetime.
Such a long-lived excited state is called "metastable".
Now we can have a population inversion of this state ($E_2$ above) with respect to the lower excited state $E_1$. 
“Pumping” can be done either electrically or optically.