Physics 228

Uncertainty Principle
Blackbody Radiation
Matter Waves
Electron Diffraction

Please join Canvas conference while listening to lecture during regular lecture period
Thursday 3/26/2020  8:55 – 9:50 AM
Heisenberg's Uncertainty Principle

The uncertainty principle states that the uncertainty of a particle’s position $\Delta x$ and the corresponding momentum uncertainty $\Delta p_x$ cannot be arbitrarily small:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

Here, $\Delta$ means “uncertainty” in position or momentum, and $\hbar = h/2\pi = 1.055 \times 10^{-34}$ J·s.

This is, of course, unlike anything encountered in classical mechanics, and quite counter-intuitive for point particles. It makes a lot more sense for waves, though!

We will illustrate how this comes about in a particular situation (photons passing through a slit), but it is true for all particles in all situations!
Wave Packets

Instead of a wave extending to positive and negative infinity, consider a wave packet of spatial extent $\Delta x$:

- The Fourier transform of such a wave packet extends in k-space by an amount $\Delta k$, with $\Delta x \Delta k \geq \frac{1}{2}$.
- The uncertainties $\Delta$ refer to the standard deviation of the probability distribution (squared wave function).
- This is a mathematical statement (about Fourier transforms), not a physical one.
- By converting wavevector $k$ to momentum via $p = \hbar k$, we get the physical uncertainty principle: $\Delta x \Delta p \geq \hbar/2$. 
Uncertainty Principle: Energy and Time

We have just seen that $p$ and $x$ are "conjugate" variables. But our equation for the traveling wave

$$E = E_0 \sin(kx - \omega t)$$
$$E = E_0 \sin(px/\hbar - Et/\hbar)$$

has two pairs of arguments, $(p, x)$ and $(E, t)$. What about localization in energy and time?

Answer: Energy - time uncertainty applies: \[\Delta E \Delta t \geq \hbar/2\]

For a laser pulse of duration $\Delta t$, the photon energy (frequency) has an uncertainty $\Delta E$.

Excited states (or particles) with a finite lifetime $\Delta t$: The excited state energy (or the particle's mass, via $E = mc^2$) is also subject to the above uncertainty.
Blackbody Radiation: What is a Blackbody?

• Blackbody radiation is the thermal radiation (light) emitted by a hot “blackbody”.

• A blackbody is an object that does not reflect or transmit any incident light. All incident light is absorbed: a cold blackbody is perfectly black. (If it is very hot it will be red, orange, yellow, or white.)

• No real materials behave in this way. A small amount of light is always reflected, even for the blackest soot or paint.

• However, we can make an artificial “black body” by considering a sufficiently large cavity in any opaque material, with a small opening.

• All light entering the opening will bounce around randomly inside the cavity, losing some of the photons at each bounce, and no light escapes back out.

• Demo: a “blackbody box”.

Blackbody Radiation

- The thermal radiation inside any cavity or box can be described as a set of standing waves or “normal modes”.

- Classical physics predicts that in thermal equilibrium, every normal mode should contain, on average, an amount of energy equal to $k_B T$ (“equipartition theorem”).

- From this we can predict the spectral energy density of the light emitted from the blackbody (cavity). The problem is, the prediction does not agree with experiment! (Total energy predicted to be infinite - oops! “Ultraviolet catastrophe”)

- If we assume, on the other hand, that each normal mode is populated by “lumps” of energy $E = hf$ (i.e., photons), where the number of such photons is determined by the rules of statistical mechanics, the correct spectral energy distribution results. Sweet.

- The formula for the spectral energy distribution emitted by a blackbody is named “Planck law”, after the guy who figured this out.
Blackbody Radiation

Wien's displacement law: \[ \lambda_{\text{max}} = \frac{b}{T} \]

Stefan-Boltzmann law: \[ j^* = \sigma T^4 \]
Quantum Weirdness

• Waves are also particles. Particles are also waves. (True for all particles, not just photons.)

• Uncertainty Principle: Precise knowledge of position and momentum of a particle, at the same time, is not possible.

• In the two-slit experiment, we cannot say which slit a photon goes through. If there is interference, the “which-slit” question is indeterminate.

• Rather than telling us what is “really” going on at the microscopic level (like: where is the particle when we are not looking at it), quantum mechanics just gives us information about the outcome of future experiments.

• Rather than telling us what will definitely happen in a given experiment, quantum mechanics only tells us the probability of certain outcomes.
Matter Waves

In 1924, Louis de Broglie proposed that, since light acts as both a particle and a wave, classical particles such as electrons also act as both particles and waves. “Matter Waves”

For matter waves, we have the same relations between momentum \( p \) and wavelength \( \lambda \) as for light:

\[
p = \frac{h}{\lambda}
\]

The uncertainty relations we discussed for photons - \( \Delta E \Delta t \geq \hbar/2, \Delta x \Delta p_x \geq \hbar/2 \), etc. also apply to particles.

But we have different relations between energy \( E \) and momentum \( p \) since the photon is massless:

- photons: \( E = pc \)
- Massive particles (relativistic): \( E^2 = (pc)^2 + (mc^2)^2 \)
- Massive particles (non-relativistic): \( K = \frac{p^2}{2m} \)
## Massless vs. Massive Particles

<table>
<thead>
<tr>
<th></th>
<th>Photons</th>
<th>Electrons, etc. (nonrelativistic)</th>
<th>Electrons, etc. (relativistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Energy</strong></td>
<td>( E = hf )</td>
<td>( E = hf )</td>
<td>( E = hf )</td>
</tr>
<tr>
<td><strong>Momentum</strong></td>
<td>( p = \frac{h}{\lambda} = \hbar k )</td>
<td>( p = mv = \frac{h}{\lambda} = \hbar k )</td>
<td>( p = \gamma mv = \frac{h}{\lambda} = \hbar k )</td>
</tr>
<tr>
<td><strong>Relativistic</strong></td>
<td></td>
<td>( p^2 = 2mK )</td>
<td>( p^2 = \left(\frac{E}{c}\right)^2 - m^2c^2 )</td>
</tr>
</tbody>
</table>

- **\( E \)**: Energy of the particle.
- **\( p \)**: Momentum of the particle.
- **\( h \)**: Planck's constant.
- **\( \lambda \)**: Wavelength.
- **\( \hbar \)**: Reduced Planck's constant.
- **\( \gamma \)**: Lorentz factor.
- **\( mv \)**: Momentum in the nonrelativistic limit.
- **\( K \)**: Kinetic energy.
Experimental Confirmation

The wave nature of electrons was demonstrated within a few years of its prediction by Davisson and Germer, by measuring diffraction of electrons by the atoms on the surface of a Ni crystal:

1. A heated filament emits electrons.
2. The electrons are accelerated by electrodes and directed at a crystal.
3. Electrons strike a nickel crystal.
4. The detector can be moved to detect scattered electrons at any angle $\theta$.
Electron Diffraction

Scattering peaks are seen at diffraction angles given by

$$m \lambda = d \sin \theta.$$  

$m$ is an integer (diffraction order)

$\lambda$ is the wavelength given by $\lambda = h/p$

d is the separation between atom rows on the surface.

At low energies, the electrons scatter off the surface, and the surface atoms act as a diffraction grating.

At high energies, the electrons penetrate the bulk of the crystal, which acts as a 3D diffraction grating, just like for x-ray diffraction in Bragg scattering:

$$m \lambda = 2d \sin \theta.$$
What is the wavelength of... 

What is the wavelength of an electron, moving at \( v = 0.9 \, \text{c} \). 

\[ p = \gamma mv = 5.64 \times 10^{-23} \, \text{kg} \cdot \text{m/s}. \]

\[ \lambda = \frac{h}{p} = 6.626 \times 10^{-34} \, \text{Js} / 5.64 \times 10^{-23} \, \text{kg} \cdot \text{m/s} \approx 10^{-11} \, \text{m}. \]

Remember the diffraction limit for imaging:

\[ \sin \theta = 1.22 \, \lambda / D. \]

Since the focal length of a lens cannot be made much smaller than the diameter, the smallest object that can be resolved in a microscope is about the size of the wavelength used.

Electron wavelengths are similar to atomic sizes, so electrons are useful for imaging atoms: Electron microscope.

The size of an atom is \( \approx 10^{-10} \, \text{m} \).

The size of an atomic nucleus is \( \approx 10^{-15} \, \text{m} \).

The size of an electron is zero (point particle, for all we know).
Electron Microscope

- Similar to optical microscope in principle, just replace light by electrons.
- For lenses, use appropriately shaped electric fields.
- Useful for imaging atoms, cells, viruses, nanostructures, ...

Diagram details:
- Vacuum chamber
- Cathode
- Accelerating anode
- Condensing lens
- Object (specimen)
- Intermediate image
- Objective lens
- Projection lens
- High-voltage supply
- Photographic film or fluorescent screen
- Eu atom
- 2 nm
- 1 μm