Physics 228

Photoelectric Effect
Energy and Momentum of Photons
Photon Absorption and Emission
Uncertainty Principle
Photoelectric Effect: Experiment

Same frequency, different intensities:

- The current is zero below a threshold voltage, independent of the intensity.
- The threshold voltage is called the “stopping potential”, as this voltage will stop the electrons’ motion and push them back toward the cathode.
- Since the electron’s kinetic energy is converted to potential energy during deceleration, the stopping potential is a measure of the initial kinetic energy.

Online simulation

The stopping potential $V_0$ is independent of the light intensity ...

... but the photocurrent $i$ for large positive $V_{AC}$ is directly proportional to the intensity.

$f$ is constant.

Constant intensity $2I$

Constant intensity $I$

$V_0$

$V_{AC}$
Photoelectric Effect: Experiment

I-V graphs for different light frequencies:

- There is a threshold frequency: no electrons are emitted when the frequency is less than some threshold.
- Once the threshold frequency is exceeded, the electrons are emitted from the material. If you change the source to higher frequency, the electrons are emitted from the material with more energy.

The stopping potential $V_0$ (and therefore the maximum kinetic energy of the photoelectrons) increases linearly with frequency: since $f_2 > f_1$, $V_{02} > V_{01}$.
From photoemission experiments, we conclude:

• Light consists of “lumps” of energy.

• The energy in each “lump” is proportional to frequency.

• Each “lump” can be emitted or absorbed all at once or not at all.

• These “lumps” thus behave as particles of light. (The light particles are more technically referred to as “photons”.)

• In the photoemission experiment, each photon, as it is absorbed, gives all its energy to an electron, which is then emitted.

• The greater the absorbed photon’s energy (frequency), the greater the photoelectron’s kinetic energy.
Energy and Mass of Photons

From the photoemission experiment we know that the energy of the photon is proportional to the frequency.

The proportionality constant is called “Planck’s constant”, denoted $h$:

$$E = hf = hc/\lambda.$$ 

From the slope we obtain $h = 6.626 \times 10^{-34}$ J·s.

What is the rest mass of a photon? Apply $E = \gamma mc^2$ to photons ($v = c$):

$$m = \frac{E}{\gamma c^2} = \frac{E}{c^2} \sqrt{1 - \left(\frac{v}{c}\right)^2} = 0$$

What is the relativistic mass of a photon?

$$m_{rel}(= \gamma m) = \frac{E}{c^2} = \frac{hf}{c^2} > 0$$

Note that $m = 0$ and $\gamma = \infty$, but the product is well defined!
Momentum of Photons

For a particle with zero rest mass the energy momentum relationship $E^2 - p^2c^2 = (mc^2)^2$ becomes $E = pc$.

The momentum of the photon is then $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$.

If we introduce the “wave number” (“wave vector“): $k = \frac{2\pi}{\lambda}$
and the “reduced Planck constant” h-bar: $\hbar = \frac{h}{2\pi}$
we can express the photon’s momentum as

\[ p = \hbar k \]
Work Function

If we apply a potential $V_0$ that just stops all the emitted electrons, we conclude that the maximum kinetic energy of the emitted electrons is:

$$K_{\text{max}} = -eV_0.$$ 

The experiment shows that the maximum kinetic energy is just the photon energy minus some offset $\phi$:

$$K_{\text{max}} = E - \phi = hf - \phi.$$ 

The downward shift of the curve $\phi$ is called the “work function” (not really a function but a constant) of the material from which the electrons are being emitted. It is typically a few electron-Volts (eV).
The work function depends on the material of the cathode:

- It represents the amount of work a photon has to do to remove a single electron from the cathode. The leftover photon energy is converted into kinetic energy of the electron.
Applications of Photoelectric Effect

• Photomultiplier tubes: ultra-sensitive light detectors, can detect single photons.
• Electron spectroscopy: Learn about the electron energy levels in different materials.
• Night vision cameras and night vision scopes.
Photoelectric Effect: Summary

The energy in electromagnetic waves is carried as indivisible lumps ("photons").

Each photon’s energy is proportional to the light frequency, with a proportionality constant $h$ ("Planck’s constant"): $E = hf$.

In the PE effect, a portion $\phi$ of the photon’s energy is used to remove an electron from the metal, while the rest is converted to kinetic energy of the electron.
Photon Absorption and Emission

Absorption can occur only when \( \Delta E = \hbar \nu = E_2 - E_1 \)

A downward transition involves emission of a photon of energy:

\[ E_{\text{photon}} = \hbar \nu = E_2 - E_1 \]

\( (\nu = f = \text{frequency}) \)

These processes cause bright emission lines and dark absorption lines in the spectra of light sources.
Spectra of Light Bulbs

(a) Continuous spectrum: light of all wavelengths is present.
(b) Line spectrum: only certain discrete wavelengths are present.

Black-body radiation

Atomic transitions
Atomic Absorption and Emission Spectra

Hydrogen

Neon

Mercury
Doppler Effect in Astrophysical Spectra

- Moving Toward You: blueshift
- At rest
- Redshifted: moving away from you
Wave-Particle Duality for Light

Wave nature of light:
- Maxwell’s equations
- Interference
- Diffraction
- Refraction
- Total internal reflection
- Brewster’s angle

Particle nature of light:
- Photoelectric effect
- Compton scattering (textbook)
- Pair production (textbook)
- Exposure of photographic film: silver grains turn dark (all or nothing)

We will see later that the same duality applies to matter (e.g., electrons and atoms).
Two-Slit Interference Revisited

• As we have seen, the interference pattern is defined by the wave nature of light.

• However, with a sensitive detector, we can see individual photons appear at distinct spots.

• Does an individual photon interfere only with other photons, or with itself?

• Can a single photon produce an interference pattern?
Two-Slit Interference Revisited

• The wave and particle descriptions are complementary: Depending on what phenomena we are studying, we will use either the wave description or the particle description.

• The square of the wave amplitude at a point in space just gives the probability that a photon will show up at that point.

• The photons (particles) appear randomly according to the probability distribution predicted by the wave nature.

• Summing over lots of photons, the wave pattern appears. (simulation)

• The probability is determined by the wave nature of light, but we nonetheless detect individual photons/particles!
Heisenberg's Uncertainty Principle

The uncertainty principle states that the uncertainty of a particle’s position $\Delta x$ and the corresponding momentum uncertainty $\Delta p_x$ cannot be arbitrarily small:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

Here, $\Delta$ means “uncertainty” in position or momentum, and $\hbar = h/2\pi = 1.055 \times 10^{-34}$ J·s.

This is, of course, unlike anything encountered in classical mechanics, and quite counter-intuitive for point particles. It makes a lot more sense for waves, though!

We will illustrate how this comes about in a particular situation (photons passing through a slit), but it is true for all particles in all situations!
Consider a photon going through a single slit of width $a$. Almost all of the light goes into a broad central maximum of width $\Theta = \lambda/a$.

Since we see photons diffracted at some non-zero angle, they have a (small) sideways component of their momentum, with a “typical” magnitude $p_y = \mu \Theta \Delta p = \mu (\lambda/a) p$. ($\mu$ is a number of order unity, which is determined by what exactly we mean by “typical”.)

The momentum of photons is related to the wavelength by $p = E/c = hf/c = h/\lambda$. We now plug this in for $p$ above.

We find $p_y = \mu(\lambda/a)(h/\lambda) = \mu h/a$, or $p_y a = \mu h$.

The “uncertainty” of the transverse momentum (resulting from going through the slit) multiplied by the “uncertainty” of the transverse position (the width of the slit) is of the order of Planck’s constant $h$. 
• If we try to localize the photon position by making a smaller slit, the phenomenon of diffraction leads to a wider central maximum, $\Theta \approx \lambda/a$, and a consequent greater variation in sideways momentum, since $p_y \approx h/a$.

• This idea is inconsistent with the classical idea that the position and the momentum of a particle can in principle both be determined exactly.

• We see that the better we determine one of these quantities, by for example making the slit smaller, the worse we determine the other.
Heisenberg's Uncertainty Principle

More formal theory (which precisely defines “typical”), results in Heisenberg's uncertainty principle:

\[ \Delta x \Delta p_x \geq \frac{\hbar}{2} \]
\[ \Delta y \Delta p_y \geq \frac{\hbar}{2} \]
\[ \Delta z \Delta p_z \geq \frac{\hbar}{2} \]

Here, \( \Delta \) means “uncertainty” in position or momentum, and \( \hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} \).

Uncertainties in different directions are unrelated. For example, \( \Delta x \Delta y \) could be 0, \( \Delta y \Delta p_z \) could be 0, and \( \Delta p_y \Delta p_z \) could also vanish.
Heisenberg's Uncertainty Principle

- As in our toy derivation, the uncertainty results from the wave nature of light.

- There is no way in principle to avoid it. This was considered in a series of thought experiments in the 1920s and 1930s, and also comes out of the formal mathematics of quantum mechanics.

- We will see later that the same uncertainty principle also applies to matter (electrons, nuclei, atoms).
Consider the light wave heading towards the slit, with electric field $E = E(x,t) = E_0 \sin(kx-\omega t)$.

This wave has energy $E = hf = \hbar \omega$, and momentum $p = \hbar k$.

This is a wave of definite momentum $p$, but we have no idea where "the photon" is located - it extends out uniformly over all $x$.

This is a "feature" of the sine wave - you pick a definite $k$ or $p$, and the wave extends uniformly over all $x$. 
Wave Packets

Instead of a wave extending to positive and negative infinity, consider a wave packet of spatial extent $\Delta x$:

- The Fourier transform of such a wave packet extends in $k$-space by an amount $\Delta k$, with $\Delta x \Delta k \geq \frac{1}{2}$.
- The uncertainties $\Delta$ refer to the standard deviation of the probability distribution (squared wave function).
- This is a mathematical statement (about Fourier transforms), not a physical one.
- By converting wavevector $k$ to momentum via $p = \hbar k$, we get the physical uncertainty principle: $\Delta x \Delta p \geq \hbar/2$. 
Uncertainty Principle: Energy and Time

We have just seen that p and x are "conjugate" variables. But our equation for the traveling wave

\[ E = E_0 \sin\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right) \]

has two pairs of arguments, (p x) and (E t). What about localization in energy and time?

Answer: Energy - time uncertainty applies:

\[ \Delta E \Delta t \geq \hbar/2 \]

This is relevant for excited states (or particles) with a finite lifetime \( \Delta t \). The excited state energy (or the particle's mass, via \( E = mc^2 \)) is then subject to the above uncertainty.