Physics 228 - Analytical Physics
SECOND COMMON HOUR EXAM
Thursday, April 10, 2014

SIGN HERE NOW:

1. The exam will last from 9:40 - 11:00 p.m. Use a #2 pencil to make entries on the answer sheet. **Enter ID information items 2-5, now, on the answer sheet before the exam starts.**
2. In the section labeled NAME (Last, First, M.I.) enter your last name, then fill in the empty circle for a blank, then enter your first name, another blank, and finally your middle initial.
3. **Under STUDENT # enter your 9-digit RUID Number.**
4. **Enter 228 under COURSE; you may ignore the section number.**
5. **Under CODE enter the exam code from the label above.**
6. During the exam, you may use pencils, a calculator, and one 8.5 x 11 inch sheet (both sides) with formulas and notes.
7. There are 15 multiple-choice questions on the exam. For each question, mark only one answer on the answer sheet. There is no deduction of points for an incorrect answer, so even if you cannot work out the answer to a question, you should make an educated guess. **At the end of the exam, hand in the answer sheet and this SIGNED cover page. Retain the exam questions attached for future reference and study.**
8. When you are asked to open the exam, make sure that your copy contains all 15 questions. Raise your hand if this is not the case, and a proctor will help you. Also raise your hand during the exam if you have a question.
9. Please have your student ID ready to show to the proctor during the exam.
Possibly useful information:

1) Speed of light in vacuum is \( c = 3.00 \times 10^8 \) m/s.
2) 1 eV = \( 1.60 \times 10^{-19} \) J.
3) Rydberg constant is \( R = 1.10 \times 10^7 \) m\(^{-1}\).
4) \( hcR = 13.60 \) eV.
5) Metric prefixes: 1 G = 10\(^9\), 1 M = 10\(^6\), 1 k = 10\(^3\), 1 m = 10\(^{-3}\), 1 \( \mu \) = 10\(^{-6}\), and 1 \( n \) = 10\(^{-9}\).
6) Planck constant is \( h = 6.63 \times 10^{-34} \) J \( \cdot \) s = 4.14 \times 10^{-15} \) eV \( \cdot \) s.
7) \( \hbar = h/2\pi = 1.05 \times 10^{-34} \) J \( \cdot \) s = 6.58 \times 10^{-16} \) eV \( \cdot \) s.
8) Electron rest mass is 9.1 \times 10^{-31} \) kg.
9) Proton rest energy is 938 MeV.
10) Bohr radius is 0.53 \times 10^{-10} \) m.
11) \( hc = 1.24 \times 10^{-6} \) eV \( \cdot \) m.
1. The ground state energy of an electron in one-dimensional infinite square potential well is 4.5 eV. What is the width of the well?
   a) 0.01 nm
   b) 0.29 nm
   c) 10.5 nm
   d) 27.8 nm
   e) 287 nm

   \[ E_g = \frac{h^2}{8mL^2} = 4.5 \times 1.6 \times 10^{-19} \text{ J} \]

   \[ \frac{(6.63 \times 10^{-34})^2}{8 \times (9.1 \times 10^{-31}) \times 4.5 \times 1.6 \times 10^{-19}} = 2.9 \times 10^{-10} \text{ m} = 0.29 \text{ nm} \]

2. A photon has an energy 1 eV. What is its wave number, \( k \)?
   a) zero, because photons are particles, not waves.
   b) \( 0.5 \times 10^7 \text{ m}^{-1} \)
   c) \( 9 \times 10^7 \text{ m}^{-1} \)
   d) \( 1.6 \times 10^{-7} \text{ m}^{-1} \)
   e) \( 3 \times 10^{-7} \text{ m}^{-1} \)

   \[ E = pc = (\frac{h}{2\pi})c \]

   \[ \Rightarrow k = \frac{E}{hc} = \frac{1 \text{ eV}}{6.68 \times 10^{-16} \text{ (eV nm)} \times 3 \times 10^8 \text{ m/s}} = 0.3 \times 10^{-8} \text{ m}^{-1} \]

3. An object moving with a relativistic speed appears to have a length (in the direction of motion) 1% smaller than its length at rest. What is the speed of the object?
   a) \( 0.99c \)
   b) \( 0.50c \)
   c) \( 0.14c \)
   d) \( 1.10c \)
   e) \( 0.01c \)

   \[ l_{av} = l_{\text{rest}} \sqrt{1 - \frac{v^2}{c^2}} \]

   \[ \Rightarrow 0.99 = \sqrt{1 - \frac{v^2}{c^2}} \]

   \[ v = c \sqrt{1 - (0.99)^2} = 0.14c \]

4. An alien spaceship escaping from the Earth fires a missile backwards (towards the Earth). The speed of the missile with respect to the ship is 0.8c. The speed of the spaceship with respect to the Earth is 0.5c. Find the speed of the missile relative to the Earth.
   a) \( 0.01c \)
   b) \( 0.3c \)
   c) \( 0.5c \)
   d) \( 0.928c \)
   e) \( c \)

   \[ u = 0.8c \]

   \[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

   \[ \frac{u + v}{1 + \frac{uv}{c^2}} = 0.8c \]

   \[ v = \frac{(0.8 - 0.5)c}{1 - 0.4} \]

   \[ \approx 0.5c \]

5. In a photo-electric effect, an electro-magnetic radiation with the wavelength 200 nm produces electrons with the kinetic energy 1.21 eV. What is the work function of the metal?
   a) 0.12 eV
   b) 13.6 eV
   c) 1.05 eV
   d) 2.8 eV
   e) 5 eV

   \[ k_{\text{max}} = \frac{h_f}{c} - \phi \]

   \[ \Rightarrow 1.21 = \frac{(4.14 \times 10^{-15}) \times (3 \times 10^8)}{(200 \times 10^{-9})} - \phi \]

   \[ \Rightarrow \phi = (1.21 - 0.21) eV = 1 eV \]
6. What is the momentum of a relativistic proton whose kinetic energy is 469 MeV?
   a) $0.16 \times 10^{-19}$ kg-m-s$^{-1}$
   b) $0.54 \times 10^{-19}$ kg-m-s$^{-1}$
   c) $1.1 \times 10^{-19}$ kg-m-s$^{-1}$
   d) $1.8 \times 10^{-19}$ kg-m-s$^{-1}$
   e) $5.59 \times 10^{-19}$ kg-m-s$^{-1}$

   $E_{\text{kin}} = \sqrt{p^2 c^2 + m^2 c^4} - mc^2$
   $\Rightarrow p^2 + (938)^2 = (469 + 938)^2$
   $\Rightarrow p = 1048 \text{ MeV}$
   $\Rightarrow p = \frac{1048 \times 1.6 \times 10^{-19} \times 10^{-6}}{3 \times 10^8} \text{ kg-m-s}^{-1}$

7. A non-relativistic free electron moving in one dimension along x is described by the wave function: $\psi(x) = A_1 e^{ix} + A_2 e^{-ix}$. What is the kinetic energy, $E$, of the electron?
   a) The kinetic energy is zero, because the electron is free.
   b) 13.6 eV
   c) $3.75 \times 10^{-20}$ J
   d) $0.06 \times 10^{-37}$ eV
   e) $3.75 \times 10^{-20}$ eV

8. An unpolarized beam of light is incident on three ideal polarizers in a row. The polarizers' transmission axes are rotated by 30 degrees with respect to the transmission axes of adjacent polarizers. What fraction of the incident light intensity is transmitted?
   a) 9/16
   b) 3/8
   c) 9/32
   d) 27/64
   e) None is transmitted, because none of the polarizer axes are parallel.

9. The kinetic energy of a relativistic particle is measured to be equal to its rest energy. What is the speed of the particle?
   a) 0
   b) 0.12c
   c) 0.668c
   d) 0.707c
   e) 0.866c

   $\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 = mc^2$
   $\Rightarrow \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = 2mc^2$
   $\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4}$
   $\Rightarrow v = c \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} c$.
10. Using uncertainty principle, estimate uncertainty in momentum of an electron moving in a one-dimensional potential well of width 1 Å (angstrom). Choose the closest answer.

\[ \Delta x \Delta p_x \geq \frac{\hbar}{2} \]

\[ \Rightarrow \Delta p_x = \frac{1.05 \times 10^{-34}}{10^{-10}} \text{ kg m s}^{-1} \]

\[ = 1.05 \times 10^{-24} \text{ kg m s}^{-1} \]

a) \( 1.05 \times 10^{-34} \text{ kg m s}^{-1} \)

b) \( 3 \times 10^6 \text{ kg m s}^{-1} \)

c) \( 1000 \text{ kg m s}^{-1} \)

d) \( 10^{-24} \text{ kg m s}^{-1} \)

e) \( 10^{24} \text{ kg m s}^{-1} \)

11. Unpolarized light is incident from a medium of refractive index \( n_1 \) onto a medium of refractive index \( n_2 \), with \( n_2 < n_1 \). The angle of incidence satisfies \( \tan(\theta) = n_2/n_1 \). Which of the following statements is correct? (Recall that the plane of incidence is the plane spanned by the incident light direction and the surface normal.)

a) The reflected light is polarized in the plane of incidence.

b) The reflected light is polarized perpendicular to the plane of incidence.

c) The transmitted light is polarized in the plane of incidence.

d) The transmitted light is polarized perpendicular to the plane of incidence.

e) There is no transmitted light because the angle of incidence is greater than the critical angle for total internal reflection.

12. An electron in a one-dimensional infinite square potential well undergoes a transition from the \( n = 2 \) excited state to the ground state and emits a photon with wavelength 400 nm. What would be the emission wavelength for the same transition in a well twice as wide?

\[ \frac{\hbar c}{n_1} = \frac{\hbar^2}{8m_1} \left( \frac{2^2 - 1^2}{2} \right) = \frac{3\hbar^2}{8m_1} \]

\[ \Rightarrow \frac{\hbar c}{n_2} = \frac{3\hbar^2}{8m_2} \Rightarrow \frac{\hbar c}{n_2} = \frac{\hbar c}{n_1} \Rightarrow \lambda_2 = 400 \cdot \left( \frac{21}{3} \right)^2 \text{ nm} \]

\[ = 1.6 \mu \text{m} \]

13. The probability to find an electron in the range \( x = 0 \) to \( 3a/4 \) in an infinite square potential well of width \( a \) is 75%. In which of the following quantum states could the electron be?

\[ P(0 < x < 3a/4) = \frac{2}{a} \int_0^{3a/4} \sin^2 \left( \frac{\pi x}{a} \right) \, dx \]

\[ = \frac{1}{a} \left[ \frac{3a}{4} \right] \left( 1 - \cos \left( \frac{\pi x}{a} \right) \right) \, dx \]

\[ = \frac{1}{a} \left( \frac{3a}{4} \right) - \frac{1}{a} \int_0^{3a/4} \cos \left( \frac{\pi x}{a} \right) \, dx \]

\[ = \frac{3}{4} - \left( \frac{1}{a} \cdot \frac{a}{2\pi} \right) \left[ \sin \left( \frac{2\pi x}{a} \cdot \frac{3a}{4} \right) \right] = 0 \]

\[ \Rightarrow \frac{3a^2}{2} = m \frac{a^2}{2} , m \text{ integer} \]

\[ \Rightarrow m = \frac{3n}{2} \]

If \( m \) is integer, \( n = 2 \).
14. A harmonic oscillator is represented by an electron attached to a spring. The ground state energy of the oscillator is 0.5 eV. What is the average kinetic energy of the oscillator in the second excited state \((n = 2)\), if we could assume that, as for a classical harmonic oscillator, the average kinetic and potential energies are equal?

\[
\begin{align*}
\text{a)} & \quad 0 \text{ eV} \\
\text{b)} & \quad 0.5 \text{ eV} \\
\text{c)} & \quad 1.6 \text{ eV} \\
\text{d)} & \quad 13.6 \text{ eV} \\
\text{e)} & \quad 1.25 \text{ eV}
\end{align*}
\]

\[
\begin{align*}
E_g &= \frac{1}{2} k \omega \\
E_2 &= (2+1) \frac{1}{2} k \omega = 5E_g = 2.5 \text{ eV} \\
\langle KE \rangle &= \frac{1}{2} E_2 = 1.25 \text{ eV}
\end{align*}
\]

15. A hydrogen atom undergoes two transitions: (1) from the state \(n = 3\) to the state \(n = 2\), and then (2) from the state \(n = 2\) to the ground state. Find the ratio of photon energies emitted in these transitions, \(E_{32}/E_{21}\).

\[
\begin{align*}
\text{a)} & \quad 1.6 \\
\text{b)} & \quad 1.05 \\
\text{c)} & \quad 0.538 \\
\text{d)} & \quad 0.124 \\
\text{e)} & \quad 0.185
\end{align*}
\]

\[
\begin{align*}
E_{32} &= 13.6 \left( \frac{1}{3} - \frac{1}{4} \right) \text{ eV} \\
E_{21} &= 13.6 \left( \frac{1}{2} - \frac{1}{4} \right) \text{ eV} \\
\frac{E_{32}}{E_{21}} &= \frac{9 - \frac{4}{36}}{9/4} = \frac{5/27}{9/4} = \frac{5}{27}
\end{align*}
\]