Final Exam:

• Tuesday Dec. 18, 4 – 7 PM

• College Avenue Gym + Annex

• Bring calculator and (up to 3) formula sheets!

• Email Prof. Montalvo if you have exam conflicts.
An inductor is connected across an ac source as shown. For this circuit, what is the relationship between the instantaneous current $i$ through the inductor and the instantaneous voltage $v_{ab}$ across the inductor?

A. $i$ is maximum at the same time as $v_{ab}$.
B. $i$ is maximum one-quarter cycle before $v_{ab}$.
C. $i$ is maximum one-quarter cycle after $v_{ab}$.
D. Two of A, B, and C are possible, depending on circumstances.
E. All three of A, B, and C are possible, depending on circumstances.
A capacitor is connected across an ac source as shown. For this circuit, what is the relationship between the instantaneous current \( i \) through the capacitor and the instantaneous voltage \( v_{ab} \) across the capacitor?

A. \( i \) is maximum at the same time as \( v_{ab} \).
B. \( i \) is maximum one-quarter cycle before \( v_{ab} \).
C. \( i \) is maximum one-quarter cycle after \( v_{ab} \).
D. Two of A, B, and C are possible, depending on circumstances.
E. All three of A, B, and C are possible, depending on circumstances.
In the transformer shown in the drawing, there are more turns in the secondary \( (N_2) \) than in the primary \( (N_1) \). In this situation, the \textit{voltage amplitude} is

A. greater in the primary than in the secondary.
B. smaller in the primary than in the secondary.
C. the same in the primary and in the secondary.
D. dependent on the frequency of the ac source.
E. dependent on the precise values of \( N_1 \) and \( N_2 \).
In the transformer shown in the drawing, there are more turns in the secondary ($N_2$) than in the primary ($N_1$). In this situation, the current amplitude is

A. greater in the primary than in the secondary.
B. smaller in the primary than in the secondary.
C. the same in the primary and in the secondary.
D. dependent on the frequency of the ac source.
E. dependent on the precise values of $N_1$ and $N_2$. 
Inductively coupled (i.e., a transformer)

Two coils are close to each other, with one connected to an alternating current source which produces a current of $I_1(t) = (5.0\, \text{A}) \sin(377t)$, while the other is connected to a resistor of $12\, \Omega$. If the current $I_2$ has a maximum of 0.40 A, what is the mutual inductance of the two coils?

a) 2.55 mH
b) 212 $\mu$H
c) 80 mH
d) 12.5 H
e) 0.96 mH

\[ |\mathcal{E}_s| = M \frac{dI_p}{dt} = I_s R_s \]

\[ M = \frac{I_s R_s}{\frac{dI_p}{dt}} = \frac{0.4 \cdot \cos(377t) \cdot 12}{377 \cdot 5 \cdot \cos(377t)} = 2.55\, \text{mH} \]
Inductively coupled (i.e., a transformer)

Two coils are close to each other, with one connected to a current source which produces a current of \( I_1(t) = 3t^2 + 5 \) amps, where \( t \) is expressed in seconds. If the mutual inductance of the two coils is 10 mH, what is the voltage \( V_2 \) across the second inductor at time \( t = 3 \) seconds?

\[
|\mathcal{E}_s| = M \frac{dI_p}{dt} = M(6t) = 10 \cdot 10^{-3} \cdot 6 \cdot 3 = 0.18V
\]

- a) 0.32 V
- b) 0.18 V
- c) 0.23 V
- d) 0.36 V
- e) 7.2 V
R-C circuit (Low-Pass Filter)

Goal: to suppress high-frequency \((f > f_0)\) components in the spectrum of a signal.

\[
V_{\text{in}} = I (R - iX_c) \quad V_{\text{out}} = I(-iX_c)
\]

\[
|V_{\text{in}}| = I \sqrt{R^2 + X_c^2} \quad |V_{\text{out}}| = IX_c
\]

\[
\left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| = \frac{X_c}{\sqrt{R^2 + X_c^2}} = \frac{1}{\omega C} = \frac{1}{\sqrt{(\omega RC)^2 + 1}} = \frac{1}{\sqrt{(\omega \tau_{RC})^2 + 1}}
\]

Output power:

\[
\left| \frac{V_{\text{out}}}{V_{\text{in}}} \right|^2 = \frac{1}{(\omega \tau_{RC})^2 + 1} \quad \begin{cases} 
1 & \omega \ll \frac{1}{\tau_{RC}} \\
1 & \omega \gg \frac{1}{\tau_{RC}} 
\end{cases}
\]

Cutoff frequency:

\[
\omega_0 = 2\pi f_0 = \frac{1}{RC}
\]

Q: How would I build a high-pass filter?
Example: R-C Circuit

A voltage from an AC power supply with $\omega = 10^3 \text{ rad/s}$ is applied across a resistor with $R = 100 \Omega$ in series with a capacitor with $C = 0.2 \ \mu\text{F}$. The current given by $i = I_{max} \cos(\omega t)$ results from an applied voltage $v = V_{max} \cos(\omega t + \phi)$ with a phase angle given by $\tan \phi =$

\begin{align*}
  &a) \ 50 \\
  &b) \ -50 \\
  &c) \ 2 \times 10^{-6} \\
  &d) \ -2 \times 10^{-6} \\
  &e) \ 1.2
\end{align*}

\[ V(t) = I(t)Z \]

\[ V_0 e^{i(\omega t + \phi)} = I_0 e^{i\omega t} |Z| e^{i\phi} \]

\[ |Z| e^{i\phi} = R - i \frac{1}{\omega C} \quad \tan \phi = -\frac{1}{\frac{\omega C}{R}} = -\frac{1}{\omega RC} \]

Note that $\phi$ is negative (as it should be for an RC circuit).

\[ \tan \phi = -\frac{1}{1 \cdot 10^3 \cdot 100 \cdot 0.2 \cdot 10^{-6}} = -50 \]
An $L-R-C$ series circuit as shown is operating at its resonant frequency. At this frequency, how are the values of the capacitive reactance $X_C$, the inductive reactance $X_L$, and the resistance $R$ related to each other?

A. $X_L = R$; $X_C$ can have any value.
B. $X_C = R$; $X_L$ can have any value.
C. $X_C = X_L$; $R$ can have any value.
D. $X_C = X_L = R$
E. None of the above is correct.
In an $L-R-C$ series circuit as shown, the current has a very small amplitude if the ac source oscillates at a very high frequency. Which circuit element causes this behavior?

A. the resistor $R$
B. the inductor $L$
C. the capacitor $C$
D. Two of these elements acting together are necessary.
E. Misleading question—the current actually has a very large amplitude if the frequency is very high.
Q31.7

In an L-R-C series circuit as shown, there is a phase angle between the instantaneous current through the circuit and the instantaneous voltage \( v_{ad} \) across the entire circuit. For what value of the phase angle is the greatest power delivered to the resistor?

A. zero  
B. 90°  
C. 180°  
D. 270°  
E. none of the above
Example: R-L-C Series Circuit

A series ac circuit is shown. The inductor has a reactance of 80 Ω and an inductance of 190 mH. A 40- Ω resistor and a capacitor whose reactance is 110 Ω, are also in the circuit. The rms current in the circuit is 2.2 A. In the figure shown, the rms voltage of the source is closest to:

\[ V_L = i \omega LI \]

\[ V_{rms\ L} = 2.2 \cdot 80 = 176V \]

\[ V_{R} \]

\[ V_{rms\ R} = 2.2 \cdot 40 = 88V \]

\[ V_{rms\ C} = 2.2 \cdot 110 = 242V \]

\[ V_C = -i \frac{I}{\omega C} \]

\[ V_{rms} = I_{rms} |Z| = I_{rms} \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \]

\[ V_{rms} = 2.2 \sqrt{40^2 + (80 - 110)^2} = 110V \]
Power in Parallel LRC Circuit

A parallel ac circuit containing a resistor with $R = 0.25\Omega$, an inductor with $L = 3.33\text{mH}$ and a capacitor with $C = 60\text{mF}$ connected to an AC voltage supply with voltage amplitude 10V and angular frequency of 100rad/s as shown in the figure. What is the average power consumption of the circuit?

\[ P_{av} = V_{rms} \cdot I_{rms} \cos \phi \]

\[ \frac{1}{Z} = \frac{1}{R} + i \left( \omega C - \frac{1}{\omega L} \right) \]

\[ \frac{1}{|Z|} = \sqrt{\frac{1}{R^2} + \left( \omega C - \frac{1}{\omega L} \right)^2} \]

\[ Z = \frac{1}{\frac{1}{R} + i \left( \omega C - \frac{1}{\omega L} \right)} = \frac{1}{\frac{1}{R}} - i \left( \omega C - \frac{1}{\omega L} \right) \]

\[ \frac{1}{\frac{1}{R}} = \frac{1}{\frac{1}{R} + i \left( \omega C - \frac{1}{\omega L} \right)} = \frac{1}{\frac{1}{R^2} + \left( \omega C - \frac{1}{\omega L} \right)^2} \]

\[ \tan \phi = \frac{-\omega C + \frac{1}{\omega L}}{1/R} = \frac{-6 + 3}{4} = -\frac{3}{4} \quad \cos \phi = \frac{1}{\sqrt{1 + \tan^2 \phi}} = \frac{4}{5} \]

\[ I_{rms} = \frac{V_{rms}}{|Z|} = V_{rms} \sqrt{\frac{1}{R^2} + \left( \omega C - \frac{1}{\omega L} \right)^2} = \frac{10}{\sqrt{2}} \sqrt{\frac{1}{0.25^2 + (6 - 3)^2}} = \frac{50}{\sqrt{2}} \text{A} \]

\[ P_{av} = V_{rms} \cdot I_{rms} \cos \phi = \frac{10}{\sqrt{2}} \cdot \frac{50}{\sqrt{2}} \cdot \frac{4}{5} = 200\text{W} \]
An \( RLC \) circuit is driven by a steady 100-Hz source with a maximum EMF of 100 V. The current in the system has maxima (amplitude) of 0.50 A. The average power dissipated in the resistor is 15 W. The phase angle between the current in the circuit and the applied EMF is

\[
P_{av} = V_{rms} \cdot I_{rms} \cos \phi = \frac{1}{2} V_0 \cdot I_0 \cos \phi
\]

\[
\cos \phi = \frac{2P_{av}}{V_0 \cdot I_0} = \frac{30W}{100V \cdot 0.5A} = 0.6
\]

\[
\arccos 0.6 \approx 0.93
\]
In a vacuum, one color of red light has a wavelength of 700 nm and one color of violet light has a wavelength of 400 nm. This means that in a vacuum, the red light

A. has higher frequency and travels faster than the violet light.
B. has higher frequency and travels slower than the violet light.
C. has lower frequency and travels faster than the violet light.
D. has lower frequency and travels slower than the violet light.
E. matches none of the above answers.
In a vacuum, one color of red light has a wavelength of 700 nm and one color of violet light has a wavelength of 400 nm. This means that in a vacuum, the red light

A. has higher frequency and travels faster than the violet light.
B. has higher frequency and travels slower than the violet light.
C. has lower frequency and travels faster than the violet light.
D. has lower frequency and travels slower than the violet light.
E. none of the above answers.
At a certain point in space, the electric and magnetic fields of an electromagnetic wave at a certain instant are given by

\[ \vec{E} = \hat{i} \left(6 \times 10^3 \text{ V/m} \right) \]
\[ \vec{B} = \hat{k} \left(2 \times 10^{-5} \text{ T} \right) \]

This wave is propagating in the

A. positive x-direction.
B. negative x-direction.
C. positive y-direction.
D. negative y-direction.
E. unknown direction.
At a certain point in space, the electric and magnetic fields of an electromagnetic wave at a certain instant are given by

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\vec{B} = \hat{k} \left( 2 \times 10^{-5} \, \text{T} \right)
\]

This wave is propagating in the

A. positive \( x \)-direction.
B. negative \( x \)-direction.
C. positive \( y \)-direction.
D. negative \( y \)-direction.
E. unknown direction.
A sinusoidal electromagnetic wave in a vacuum is propagating in the positive $z$-direction. At a certain point in the wave at a certain instant in time, the electric field points in the negative $x$-direction. At the same point and at the same instant, the magnetic field points in the

A. positive $y$-direction.
B. negative $y$-direction.
C. positive $z$-direction.
D. negative $z$-direction.
E. unknown direction.
A sinusoidal electromagnetic wave in a vacuum is propagating in the positive $z$-direction. At a certain point in the wave at a certain instant in time, the electric field points in the negative $x$-direction. At the same point and at the same instant, the magnetic field points in the

A. positive $y$-direction.
B. negative $y$-direction. \[\checkmark\]
C. positive $z$-direction.
D. negative $z$-direction.
E. unknown direction.
In a sinusoidal electromagnetic wave in a vacuum, the electric field has only an $x$-component. This component is given by

$$E_x = E_{\text{max}} \cos (ky + \omega t)$$

This wave propagates in the

A. positive $z$-direction.
B. negative $z$-direction.
C. positive $y$-direction.
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E. unknown direction.
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This wave propagates in the

A. positive $z$-direction.
B. negative $z$-direction.
C. positive $y$-direction.
D. negative $y$-direction.
E. unknown direction.
In a sinusoidal, traveling electromagnetic wave in a vacuum, the magnetic energy density

A. is the same at all points in the wave.
B. is maximum where the electric field has its greatest magnitude.
C. is maximum where the electric field is zero.
D. is negative where the electric energy density is positive.
E. More than one of the above is true.
In a sinusoidal, traveling electromagnetic wave in a vacuum, the *magnetic* energy density

A. is the same at all points in the wave.

B. is maximum where the *electric* field has its greatest magnitude.

C. is maximum where the *electric* field is zero.

D. is negative where the electric energy density is positive.

E. More than one of the above is true.
The drawing shows a sinusoidal electromagnetic wave in a vacuum at one instant of time at points between $x = 0$ and $x = \lambda$.

At this instant, at which of the values of $x$ in the drawing does the instantaneous Poynting vector have its maximum magnitude?

A. $x = 0$ and $x = \lambda$ only
B. $x = \lambda/4$ and $x = 3\lambda/4$ only
C. $x = \lambda/2$ only
D. $x = 0$, $x = \lambda/2$, and $x = \lambda$
E. none of the above
The drawing shows a sinusoidal electromagnetic wave in a vacuum at one instant of time at points between $x = 0$ and $x = \lambda$.

At this instant, at which of the values of $x$ in the drawing does the instantaneous Poynting vector have its maximum magnitude?

A. $x = 0$ and $x = \lambda$ only  
B. $x = \lambda/4$ and $x = 3\lambda/4$ only  
C. $x = \lambda/2$ only  
D. $x = 0$, $x = \lambda/2$, and $x = \lambda$  
E. none of the above
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the net magnetic force on the loop

A. is perpendicular to the plane of the loop, in a direction given by a right-hand rule.
B. is perpendicular to the plane of the loop, in a direction given by a left-hand rule.
C. is in the same plane as the loop.
D. is zero.
E. depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the *net magnetic force* on the loop

- A. is perpendicular to the plane of the loop, in a direction given by a right-hand rule.
- B. is perpendicular to the plane of the loop, in a direction given by a left-hand rule.
- C. is in the same plane as the loop.
- D. is zero.
- E. depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the net magnetic torque on the loop

A. tends to orient the loop so that its plane is perpendicular to the direction of the magnetic field.
B. tends to orient the loop so that its plane is edge-on to the direction of the magnetic field.
C. tends to make the loop rotate around its axis.
D. is zero.
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the *net magnetic torque* on the loop

A. tends to orient the loop so that its plane is perpendicular to the direction of the magnetic field.

B. tends to orient the loop so that its plane is edge-on to the direction of the magnetic field.

C. tends to make the loop rotate around its axis.

D. is zero.
Q28.4

Two long, straight wires are oriented perpendicular to the \(xy\)-plane. They carry currents of equal magnitude \(I\) in opposite directions as shown. At point \(P\), the magnetic field due to these currents is in

A. the positive \(x\)-direction.
B. the negative \(x\)-direction.
C. the positive \(y\)-direction.
D. the negative \(y\)-direction.
E. none of the above.
Two long, straight wires are oriented perpendicular to the $xy$-plane. They carry currents of equal magnitude $I$ in opposite directions as shown. At point $P$, the magnetic field due to these currents is in

A. the positive $x$-direction.
B. the negative $x$-direction.
C. the positive $y$-direction.
D. the negative $y$-direction.
E. none of the above.
The long, straight wire $AB$ carries a 14.0-A current as shown. The rectangular loop has long edges parallel to $AB$ and carries a clockwise 5.00-A current. What is the direction of the net magnetic force that the straight wire $AB$ exerts on the loop?

A. to the right
B. to the left
C. upward (toward $AB$)
D. downward (away from $AB$)
E. Misleading question—the net magnetic force is zero.
The long, straight wire $AB$ carries a 14.0-A current as shown. The rectangular loop has long edges parallel to $AB$ and carries a clockwise 5.00-A current. What is the direction of the net magnetic force that the straight wire $AB$ exerts on the loop?

A. to the right  
B. to the left  
C. upward (toward $AB$)  
D. downward (away from $AB$)  
E. Misleading question—the net magnetic force is zero.
A wire consists of two straight sections with a semicircular section between them. If current flows in the wire as shown, what is the direction of the magnetic field at $P$ due to the current?

A. to the right
B. to the left
C. out of the plane of the figure
D. into the plane of the figure
E. Misleading question—the magnetic field at $P$ is zero.
A wire consists of two straight sections with a semicircular section between them. If current flows in the wire as shown, what is the direction of the magnetic field at $P$ due to the current?

A. to the right
B. to the left
C. out of the plane of the figure
D. into the plane of the figure
E. Misleading question—the magnetic field at $P$ is zero.

\[
\vec{B}(r) = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}
\]
A circular loop of wire is in a region of spatially uniform magnetic field. The magnetic field is directed into the plane of the figure. If the magnetic field magnitude is decreasing,

A. the induced emf in the loop is clockwise.
B. the induced emf in the loop is counterclockwise.
C. the induced emf in the loop is zero.
D. either A or B is possible.
E. any of A, B, or C is possible.
A circular loop of wire is in a region of spatially uniform magnetic field. The magnetic field is directed into the plane of the figure. If the magnetic field magnitude is *decreasing*,

A. the induced emf in the loop is clockwise.
B. the induced emf in the loop is counterclockwise.
C. the induced emf in the loop is zero.
D. either A or B is possible.
E. any of A, B, or C is possible.
A flexible loop of wire lies in a uniform magnetic field of magnitude $B$ directed into the plane of the picture. The loop is pulled as shown, reducing its area. The induced current flows

A. downward through resistor $R$ and is proportional to $B$.
B. upward through resistor $R$ and is proportional to $B$.
C. downward through resistor $R$ and is proportional to $B^2$.
D. upward through resistor $R$ and is proportional to $B^2$.
E. None of the above is true.
A flexible loop of wire lies in a uniform magnetic field of magnitude $B$ directed into the plane of the picture. The loop is pulled as shown, reducing its area. The induced current flows

A. downward through resistor $R$ and is proportional to $B$.
B. upward through resistor $R$ and is proportional to $B$.
C. downward through resistor $R$ and is proportional to $B^2$.
D. upward through resistor $R$ and is proportional to $B^2$.
E. None of the above is true.
The drawing shows the uniform magnetic field inside a long, straight solenoid. The field is directed into the plane of the drawing and is increasing. What is the direction of the electric force on a positive point charge placed at point $b$?

A. to the left  
B. to the right  
C. straight up  
D. straight down  
E. Misleading question—the electric force at this point is zero.
The drawing shows the uniform magnetic field inside a long, straight solenoid. The field is directed into the plane of the drawing and is increasing. What is the direction of the electric force on a positive point charge placed at point $b$?

A. to the left
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C. straight up
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E. Misleading question—the electric force at this point is zero.
The drawing shows the uniform magnetic field inside a long, straight solenoid. The field is directed into the plane of the drawing and is increasing. What is the direction of the **magnetic** force on a positive point charge placed at point \( a \)?

A. to the left  
B. to the right  
C. straight up  
D. straight down  
E. zero
The drawing shows the uniform magnetic field inside a long, straight solenoid. The field is directed into the plane of the drawing and is increasing. What is the direction of the magnetic force on a positive point charge placed at point $a$?

A. to the left
B. to the right
C. straight up
D. straight down
E. zero
A circular loop of copper wire is placed next to a long, straight wire. The current \( I \) in the long, straight wire is increasing. What current does this induce in the circular loop?

A. a clockwise current  
B. a counterclockwise current  
C. zero current  
D. either A or B  
E. any of A, B, or C
A circular loop of copper wire is placed next to a long, straight wire. The current $I$ in the long, straight wire is increasing. What current does this induce in the circular loop?

A. a clockwise current
B. a counterclockwise current
C. zero current
D. either A or B
E. any of A, B, or C
A circular loop of wood is placed next to a long, straight wire. The resistivity of wood is about $10^{20}$ times greater than that of copper. The current $I$ in the long, straight wire is increasing. Compared to the emf that would be induced if the loop were made of copper, the emf induced in the loop of wood is

A. about $10^{-20}$ as great.
B. about $10^{-10}$ as great.
C. about $10^{-5}$ as great.
D. the same.
E. greater.
A circular loop of wood is placed next to a long, straight wire. The resistivity of wood is about $10^{20}$ times greater than that of copper. The current $I$ in the long, straight wire is increasing. Compared to the emf that would be induced if the loop were made of copper, the emf induced in the loop of wood is

A. about $10^{-20}$ as great.
B. about $10^{-10}$ as great.
C. about $10^{-5}$ as great.
D. the same.
E. greater.
The rectangular loop of wire is being moved to the right at constant velocity. A constant current $I$ flows in the long, straight wire in the direction shown. The current induced in the loop is

A. clockwise and proportional to $I$.
B. counterclockwise and proportional to $I$.
C. clockwise and proportional to $I^2$.
D. counterclockwise and proportional to $I^2$.
E. zero.
The rectangular loop of wire is being moved to the right at constant velocity. A constant current $I$ flows in the long, straight wire in the direction shown. The current induced in the loop is

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B. counterclockwise and proportional to $I$.
C. clockwise and proportional to $I^2$.
D. counterclockwise and proportional to $I^2$.
E. zero.
Good Luck!!