Outline:

- Transfer of electromagnetic energy in space: Poynting vector
- Radiation pressure.
Intensity of Electromagnetic Waves

Energy density of EM wave:

\[ u_{EB}(r, t) = \varepsilon_0 E(r, t)^2 \]

Usually we are interested in the *space-averaged* or *time-averaged* quantities:

\[ \langle \cos^2(x) \rangle = \left\langle \frac{1}{2} + \frac{1}{2} \cos(2x) \right\rangle = \frac{1}{2} \]

Write energy density in terms of \( E_0 \), the *amplitude* of \( E(r, t) = E_0 \cos(kx - \omega t) \):

\[ \langle u_{EB} \rangle = \varepsilon_0 \langle E^2(r, t) \rangle = \varepsilon_0 E_0^2 \langle \cos^2(kx - \omega t) \rangle = \frac{\varepsilon_0}{2} E_0^2 \]

**Intensity:** The average *power* transported by the EM wave per unit area:

\[ I = \frac{U}{\Delta t \cdot A} = \frac{\langle u \rangle \Delta \text{vol}}{\Delta t \cdot A} = \frac{\langle u \rangle A \Delta x}{\Delta t \cdot A} = c \langle u \rangle \]

Intensity:

\[ I = c \langle u_{EB} \rangle = \frac{1}{2} c \varepsilon_0 E_0^2 = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 = \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} B_0^2 \]

Units: W/m²
Imagine that when you switch on your lamp, you increase the intensity of light shining on your textbook by a factor of 16. By what factor does the electric field amplitude of this light wave increase?

A. 256
B. 16
C. 4
D. 2
E. 1
Can we express the energy flow in terms of the electric and magnetic fields?

Yes: define the **Poynting vector**:  
\[
\vec{S} = \frac{1}{\mu_0} [\vec{E} \times \vec{B}]
\]

\[
\langle S \rangle = \left( \frac{1}{\mu_0} E_0 B_0 \cos^2 \omega t \right) = \frac{1}{2\mu_0} E_0 B_0
\]

For EM waves, this is just the intensity \( I \) (= energy flux).

It turns out that this is true more generally: Whenever E and B fields are present at the same point in space and time, the **Poynting vector** represents the energy flux at that point!
Example: Transmission Line

The power dissipated in the resistor is $P = VI$. The power provided by the battery is also $P = VI$. Somehow the energy flows from the battery to the resistor. How does it get there?

According to Poynting, the energy flows through the fields, not the wires!

To simplify the math, instead of wires, let’s run the current $I$ through two metal ribbons of width $W$, separated by $a << W$.

Using the Poynting vector formalism, calculate the power transmitted to the load $R$:

Transmitted power:

$$P = \int_{\text{cross section}} \vec{S} \cdot d\vec{A} = \int \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \cdot d\vec{A} = \frac{1}{\mu_0} EBA$$

$$E = \frac{V}{a} \quad B = \mu_0 \frac{I}{W} \quad \text{E and B uniform, orthogonal to one another}$$

$$BW = \mu_0 I$$

$$P = \frac{1}{\mu_0} \frac{V}{a} \mu_0 \frac{I}{W} aW = VI$$
Let’s consider (slowly) charging a capacitor:

Electric field energy builds up between the plates. Since the Poynting vector “poynts” radially inward, the energy isn’t actually coming in along the wires, but rather from the field surrounding the capacitor!

Electric field energy:

\[ U_E = \text{volume} \cdot u_E = (\pi a^2 \cdot h) \frac{1}{2} \epsilon_0 E^2 \]

Charging power:

\[ \frac{dU_E}{dt} = (\pi a^2 \cdot h) \epsilon_0 E \frac{dE}{dt} \]

Use Ampere’s law to determine \( B \) at radius \( a \):

\[ \oint_{\text{loop}} \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \int_{\text{surf}} \frac{d\vec{E}}{dt} \cdot d\vec{A} \]

\[ B \cdot 2\pi a = \mu_0 \epsilon_0 \frac{dE}{dt} \pi a^2 \]

\[ B = \frac{1}{2} \mu_0 \epsilon_0 a \frac{dE}{dt} \]

Energy flow (power) from outside through cylindrical surface of radius \( a \) and height \( h \):

\[ S \cdot 2\pi ah = \frac{1}{\mu_0} EB \cdot 2\pi ah = \frac{1}{\mu_0} E \cdot \frac{1}{2} \mu_0 \epsilon_0 a \frac{\partial E}{\partial t} \cdot 2\pi ah = (\pi a^2 \cdot h) \epsilon_0 E \frac{dE}{dt} \]

Same as charging power!
Standing Waves

- A **standing wave** is a superposition of a forward traveling wave and a reflected wave.
- Standing waves have stationary nodes and antinodes (e.g., guitar strings).
- Standing waves oscillate in time, but do not travel.
- A standing EM wave consists of a standing electric field wave and a standing magnetic field wave.
- The electric field has its nodes where the magnetic field has its antinodes, and vice versa.
- In time, E and B also oscillate *out of phase.*
Standing Waves in a Microwave Oven

- Heating occurs only around the antinodes of the electric field.
- To ensure uniform heating, a spinning turntable moves the food around.
The *instantaneous* Poynting vector of the standing wave shown in the picture

A. points along the $x$-axis.
B. points along the $y$-axis.
C. points along the $z$-axis.
D. is always zero.
E. none of the above

\[ \vec{E} = 2E_0 \cos(kx) \sin(\omega t) \hat{j} \]
\[ \vec{B} = 2B_0 \sin(kx) \cos(\omega t) \hat{k} \]

\[ \hat{S} = \frac{1}{\mu_0} [\vec{E} \times \vec{B}] \]
The *time-averaged* Poynting vector of the standing wave shown in the picture:

A. points along the $x$-axis.
B. points along the $y$-axis.
C. points along the $z$-axis.
D. is zero.
E. none of the above

\[
\vec{E} = 2E_0 \cos(kx) \sin(\omega t) \hat{j} \\
\vec{B} = 2B_0 \sin(kx) \cos(\omega t) \hat{k}
\]

\[
\vec{S} = \frac{1}{\mu_0} [\vec{E} \times \vec{B}]
\]
At a certain point in space, the electric and magnetic fields of a traveling electromagnetic wave at a certain instant are given by

\[
\vec{E} = \hat{i}(6 \times 10^3 \text{ V/m}) \quad \vec{B} = \hat{k}(2 \times 10^{-5} \text{ T})
\]

This wave is propagating in the

A. positive x-direction.  
B. negative x-direction.  
C. positive y-direction.  
D. negative y-direction.  
E. none of the above
A chunk of electromagnetic field energy \( U = m_{rel}c^2 \), has a **relativistic mass** of \( m_{rel} = U/c^2 \).

The corresponding **momentum** carried by a wave traveling at speed \( c \) is then \( m_{rel}c = U/c \).

According to Newton’s 2\(^{nd} \) law, when the wave is absorbed, it exerts a force

\[
F_{rad} = \frac{d}{dt} (m_{rel}c) = \frac{d}{dt} \left(\frac{U}{c}\right) = \frac{1}{c} \frac{dU}{dt}
\]

called the **radiation pressure** force.

The radiation pressure is the force per unit area:

\[
P_{rad} = \frac{F_{rad}}{A} = \frac{1}{c} \frac{dU}{dt} \frac{I}{c} = \frac{I}{c} = \langle u_{EB} \rangle
\]

(assuming normal incidence)

We may write this in terms of the magnitude of the averaged Poynting vector:

\[
P_{rad} = \frac{S_{avg}}{c}
\]

If, instead of being absorbed, the wave is normally **reflected**, the momentum change is twice as large, resulting in **twice** the radiation pressure:

\[
P_{rad} = \frac{2I}{c} = \frac{2S_{avg}}{c}
\]
Radiation pressure is responsible for the tails of comets:

Find the pressure of sunlight at the earth’s surface, using intensity \( S = 1,000 \text{W/m}^2 \) and assuming that the radiation is absorbed.

\[
P = \frac{S}{c} = \frac{1,000 \text{W/m}^2}{3 \times 10^8 \text{m/s}} \approx 3 \times 10^{-6} \text{Pa}
\]
Interstellar spacecraft could be propelled by solar radiation pressure pushing gigantic sails:

Even alien spacecraft from other stars...

**COULD SOLAR RADIATION PRESSURE EXPLAIN ‘OUMUAMUA’S PECULIAR ACCELERATION?**

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We have shown that the observed non-gravitational acceleration of ‘Oumuamua, may be explained by Solar radiation pressure. This requires a small mass-to-area ratio for ‘Oumuamua of \((m/A) \approx 0.1 \text{ g cm}^{-2}\). For a planar geometry and typical mass densities of 1–3 g cm\(^{-2}\) this gives an effective thickness of only 0.9–0.3 mm, respectively. For a mate-

Alternatively, a more exotic scenario is that ‘Oumuamua may be a fully operational probe sent *intentionally* to Earth vicinity by an alien civilization. Based on the PAN-STARRS1