Lecture 14: Magnetic Forces on Currents.

Outline:
- Magnetic Force on a Wire Segment.
- Hall Effect.
- Torque on a Current-Carrying Loop.

Lecture 13 review:
Magnetic Forces on Moving Charges

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]
Under what circumstances is the total magnetic flux through a closed surface *positive*?

A. If the surface encloses the north pole of a magnet, but not the south pole.

B. If the surface encloses the south pole of a magnet, but not the north pole.

C. If the surface encloses both the north and south poles of a magnet.

D. Never.

E. None of the above.
A particle with a positive charge moves in the $xz$-plane as shown. The magnetic field is in the positive $z$-direction. The magnetic force on the particle is in

A. the positive $x$-direction.
B. the negative $x$-direction.
C. the positive $y$-direction.
D. the negative $y$-direction.
E. none of these
Consider a wire of length $l$ and cross-sectional area $A$:

The Lorentz force on each charge carrier is

$$
\vec{f} = q(\vec{v}_d \times \vec{B}).
$$

The total magnetic force on all charge carriers is obtained by summing over charge carriers:

$$
\vec{F} = Q(\vec{v}_d \times \vec{B}), \quad \text{where} \quad Q = nqAl.
$$

Thus

$$
\vec{F} = nqAl(\vec{v}_d \times \vec{B}) = nqAv_d(\vec{l} \times \vec{B})
$$

Remember that the current

$$
I = nqv_dA.
$$

The magnetic force on the wire is then:

$$
\vec{F} = I(\vec{l} \times \vec{B})
$$
A charged particle moves through a region of space that has both a uniform electric field and a uniform magnetic field. In order for the particle to move through this region at a constant velocity,

A. the electric and magnetic fields must point in the same direction.

B. the electric and magnetic fields must point in opposite directions.

C. the electric and magnetic fields must point in perpendicular directions.

D. The answer depends on the sign of the particle’s electric charge.
Hall Effect

Consider the force on the electrons in more detail:

In the steady state, the magnetic force on moving charges is compensated by the electrostatic force due to *uncompensated* surface charges:

$$\vec{F}_E = -\vec{F}_B$$  \hspace{1cm}  $$qE = qv_d B$$  \hspace{1cm}  $$E = v_d B$$

$$I = qnv_d W t$$  \hspace{1cm}  $$n$$ – the density of mobile carriers  
$$W$$ – the width of the conductor  
$$t$$ – the thickness of the conductor

$$V = EW = v_d BW = \frac{IBW}{qnWt}$$  \hspace{1cm}  Hall voltage  
$$V_H = \frac{IB}{qnt}$$

The sign of $V_H$ depends on the sign of the charge of current carriers $q$, the magnitude is proportional to $1/n$. 

Edwin Hall  
(1855-1938)
Example: typical two-dimensional \((n \cdot t)\) charge density in Si field-effect transistor (in the “on” state) is \(10^{16} \text{ 1/m}^2\). Let’s run a 0.1A current and place the transistor in a 0.1T magnetic field:

\[
V_H = \frac{IB}{qnt} = \frac{0.1A \cdot 0.1T}{1.6 \cdot 10^{-19}C \cdot 1 \cdot 10^{16} m^{-2}} \approx 6V
\]

Use as a magnetic field sensor (Hall sensor):
Example: a straight horizontal length of wire has a mass of \(m/l=10 \text{ g/m}\); it carries a current of 1A. What are the magnitude and direction of the minimum magnetic field needed to suspend the wire in the Earth’s gravitational field?

\[
F_B = mg \quad lIB = \left(\frac{m}{l}\right)lg \quad B = \frac{(m/l)g}{I} = \frac{0.01 \text{ kg/m} \cdot 10 \text{ m/s}^2}{1 \text{ A}} = 0.1T
\]
Ampere’s Motor

Which orientation?

A

B

Switch

Conducting bar

Conducting rails

\[ \vec{F} \quad \vec{B} \]
Dipoles in Uniform Fields

Net force = 0, torque ≠ 0
\[ \hat{\tau} = \vec{r} \times \vec{F} \]

**Electric**

\[ \vec{p} = q_+ (\vec{r}_+ - \vec{r}_-) \]
\[ \hat{\tau} = \vec{p} \times \vec{E} \]

\[ U(\phi) = -pE \cos(\phi) \]

**Magnetic**

\[ F = ILB \]
Torque on a Current Loop

Consider \( \vec{A} \perp \vec{B} \) (\( \vec{B} \) in the loop’s plane): \( F_1 = F_2 = aIB \quad F_3 = F_4 = 0 \)

\[
\begin{align*}
\vec{\tau} &= \frac{b}{2} \hat{z} \times \vec{F}_1 + \frac{b}{2} (\hat{z}) \times \vec{F}_2 \\
&= \frac{b}{2} aIB(-\hat{y}) + \frac{b}{2} aIB(-\hat{y}) = abIB(-\hat{y})
\end{align*}
\]

**Magnetic dipole moment:**

\[ \mu = abI = A I \quad (A \text{ – the loop’s area}) \]

Direction of \( \vec{\mu} \): the right-hand rule.

In general: \( \vec{\tau} = \vec{\mu} \times \vec{B} \)

\( \tau = \mu B \sin \phi \)

(compare with \( \vec{\tau} = \vec{p} \times \vec{E} \))

If there are \( N \) turns, the total magnetic dipole moment is \( \mu = NAI \)

\( \tau = 0 \) if \( \phi = 0^0, 180^0 \)
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the net magnetic torque on the loop

A. tends to orient the loop so that its plane is perpendicular to the direction of the magnetic field.
B. tends to orient the loop so that its plane is edge-on to the direction of the magnetic field.
C. tends to make the loop rotate around its axis.
D. is zero.
E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
Magnetic Dipole vs. Electric Dipole

**Similarity:**
- a magnet’s magnetic field is very similar to a dipole’s electric field *at points far from the dipoles*;
- both repel/attract each other;
- both align along the field lines.

**Difference:**
- unlike electric dipoles, magnetic poles cannot be separated;
- magnets have no effect on stationary charges.
The Earth's North Magnetic Pole is actually a magnetic *south* pole and the Earth's South Magnetic Pole is a magnetic *north* pole.
Magnetic Dipole in Non-Uniform Magnetic Field

\[ F_z = \frac{d}{dz} \mu \cdot B \]

Recall: induced electric dipoles are oriented along the electric field, they are always \textit{attracted} to the region of a stronger field:

\[ F_z = \frac{d}{dz} p \cdot E \]

Paramagnetic response: the induced \( \vec{\mu} \) is parallel to \( \vec{B} \)

Diamagnetic response: the induced \( \vec{\mu} \) is anti-parallel to \( \vec{B} \)

Levitation of a diamagnetic object

Andre Geim
Nobel 2010
IgNobel 2000

(No frogs were hurt in the production of this video!)