What are **magnetic forces**?

- Forces between magnets
- Forces between magnets and magnetizable materials
- Forces between magnets and *moving* charges (currents)
- Forces between *moving* charges (currents)
- Mediated by a “magnetic field” \( B \).
Characteristic Magnetic Fields

Units: tesla, T

\[ B \approx 10^{-4}T \]

\[ B \approx 1 - 2T \]

Superconducting solenoids in MRI scanners

\[ B \text{ up to } 8T \]

Superconducting solenoids in the LHC

\[ B \text{ up to } 30T \]

Superconducting solenoids

rare-earth magnets

\[ B \text{ up to } 1.4T \]
Sources of Magnetic Field

Charges at rest do not generate magnetic fields $B$. What are the sources of the magnetic field?

- “Magnetic point charges” (magnetic monopoles)? Have never been observed - though their existence doesn’t contradict any fundamental laws of physics.
- Time-dependent electric fields: we’ll consider this later in electrodynamics.
- Charges in motion: currents, orbital motion of electrons in atoms.

In magnetostatics:

- $B$ field independent of time
- Steady-state (time-independent) currents.
Magnetic Field Lines

Magnetic field lines are *closed loops* (no magnetic monopoles):

Magnetic field is a *non-conservative* vector field.

\[ \oint \mathbf{B} \cdot d\mathbf{l} \neq 0 \]

to be specified later
Flux of the Magnetic Field & Gauss’ Law

Just like for the electric field, we define the **magnetic flux** through a surface:

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

**Units**: T·m² = Weber, Wb

Compare with electric flux:

\[ \Phi_E = \int \vec{E} \cdot d\vec{A} \]

Electric field lines start/end at charges

Magnetic field lines form closed loops

Gauss’ law for closed surfaces:

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \]

\[ \oint \vec{B} \cdot d\vec{A} = 0 \]
Force on a Charge Moving in Magnetic Field

Charges moving in a magnetic field experience a **transverse force**:

**Lorentz force:**
\[ \mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \]

**Magnitude:**
\[ F = qvB\sin\phi \]

Right-hand rule (for **positive** charges)
Force on a Charge Moving in Magnetic Field

Lorentz force

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]

\( \vec{F} = 0 \) for charges moving along \( \vec{B} \)

\( F \) is max when \( \vec{v} \perp \vec{B} \)

Compare with electrostatic force:

\[ \vec{F} = q\vec{E} \] (defines the electric field)

\[ \vec{F} = q(\vec{v} \times \vec{B}) \] (defines the magnetic field)

Units: Tesla

\[ T = \frac{N}{C \frac{m}{s}} = \frac{Ns}{Cm} = \frac{N}{Am} \]

Another unit: Gauss

\[ G = 10^{-4}T \]
Imagine that you are looking at the face of a CRT. The bright spot indicating where the electron beam hits the face. You bring a permanent magnet toward the CRT with its north pole oriented upward. Which direction will the spot be deflected across the screen?

A. up  \[ \vec{F} = q(\vec{v} \times \vec{B}) \]
B. down
C. the spot does not deflect
D. right
E. left
When does a magnetic field exert a force on a charged particle?

A. Always.

B. Only when the particle moves exactly perpendicular to the magnetic field lines.

C. When the particle is moving at a non-zero angle with respect to the magnetic field lines.

D. When the particle is moving along the magnetic field lines.

E. When the particle is moving.

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]
\[ \mathbf{F} = q(\mathbf{\dot{v}} \times \mathbf{B}) \quad \Rightarrow \quad \mathbf{F} \perp d\mathbf{l} \quad \Rightarrow \quad dW = \mathbf{F} \cdot d\mathbf{l} = 0 \]

The work done by the magnetic field on a moving charge = 0

You cannot *increase* the speed of charged particles using a magnetic field.

The \( B \)-induced acceleration is not zero, it’s perpendicular to the velocity \( \mathbf{a} = \frac{d\mathbf{v}}{dt} \).
When a charged particle moves through a magnetic field, the trajectory of the particle at a given point is

A. parallel to the magnetic field line that passes through that point.

B. perpendicular to the magnetic field line that passes through that point.

C. neither parallel nor perpendicular to the magnetic field line that passes through that point.

D. any of the above, depending on circumstances
An electron moves along the z axis. The $B$ field is in the $x$-$y$ plane.

\[ \vec{v} = 1\hat{k} \text{ m/s} \quad \vec{B} = (2\hat{i} + 3\hat{j})T \]

What is the Lorentz force on the electron?

\[ \vec{F} = -e(\vec{v} \times \vec{B}) = -e \left( 1\hat{k} \times (2\hat{i} + 3\hat{j}) \right) \]

\[ = -e \left( 1\hat{k} \times 2\hat{i} + 1\hat{k} \times 3\hat{j} \right) \]

\[ = -e(2\hat{j} - 3\hat{i}) = e(3\hat{i} - 2\hat{j}) \]
A particle with charge $q = -1 \text{ C}$ is moving in the positive $z$-direction at 5 m/s. The magnetic field at its position is

$$\vec{B} = (3\hat{i} - 4\hat{j}) \text{ T}$$

What is the magnetic force on the particle?

A. $(20\hat{i} + 15\hat{j}) \text{ N}$  
B. $(20\hat{i} - 15\hat{j}) \text{ N}$  
C. $(-20\hat{i} + 15\hat{j}) \text{ N}$  
D. $(-20\hat{i} - 15\hat{j}) \text{ N}$  
E. none of these

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]
A positively charged particle moves in the positive $z$-direction. The magnetic force on the particle is in the positive $y$-direction. What can you conclude about the $z$-component of the magnetic field at the particle’s position?

A. $B_z > 0$
B. $B_z = 0$
C. $B_z < 0$
D. Nothing much

\[ \mathbf{F} = q \mathbf{v} \times \mathbf{B} \]

\[ \hat{i} \times \hat{j} = \hat{k} \]
\[ \hat{j} \times \hat{k} = \hat{i} \]
\[ \hat{k} \times \hat{i} = \hat{j} \]
The Lorentz force causes charged particles to move in a circular trajectory:

$$\vec{F} = q(\vec{v} \times \vec{B}) \rightarrow qvB = m \frac{v^2}{R}$$

$$R = \frac{mv}{qB}$$

By measuring $R$ and $v$, one can determine the ratio $m/q$.

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

The period $T$ is independent of $v$!

If a charge has a velocity component along $\vec{B}$, get a helical trajectory:
What’s wrong? In this form, the equation works only for non-relativistic motion. To correct, you need to replace the rest mass $m$ with the relativistic mass $m/\sqrt{1 - (v/c)^2}$. 

Given:

- $m \approx 1.6 \cdot 10^{-27} \text{kg}$
- $v \approx c$
- $B = 8 \text{T}$

The calculation is:

$$R = \frac{mv}{qB}$$

$$R = \frac{1.6 \cdot 10^{-27} \text{kg} \cdot 3 \cdot 10^8 \text{m/s}}{1.6 \cdot 10^{-19} \text{C} \cdot 8 \text{T}} \approx 0.38 \text{m}$$

And the circumference of the Large Hadron Collider is approximately 8.6 km.
Mass Spectrometer

\[ \frac{mv^2}{2} = qV \]
\[ v = \sqrt{\frac{2qV}{m}} \]

\[ qE = qvB \]
\[ v = \frac{E}{B} \]

\[ R = \frac{mv}{qB} \]