How long does it take to charge the capacitor?

Clearly, the charging current is limited by the resistance:
- Internal resistance of power supply
- Resistance of wires & switch

So let’s explicitly insert a resistor!
**Capacitor Charging (qualitative)**

**Initial state** (switch open): \( i=0, \; q=0, \; V_{ab}=0, \; V_{bc}=0 \)

**At t=0, we close the switch:** \( q(0)=0, \; V_{bc}(0)=0 \)

\[
q(t) = V_{bc}(t)C
\]

\[
\mathcal{E}C
\]

\[
i(t) = \frac{dq(t)}{dt}
\]

\[
\mathcal{E}/R
\]
Charging a Capacitor

**Initial state** \( t = 0 \): \( i=0, \ q=0, \ V_{ab}=0, \ V_{bc}=0 \)

**At \( t=0 \), we close the switch:**

\[
\mathcal{E} - i(t)R - \frac{q(t)}{C} = 0
\]

\[
i(t) = \frac{dq(t)}{dt}
\]

\[
\frac{dq(t)}{dt} + \frac{q(t)}{RC} = \frac{\mathcal{E}}{R}
\]

What are the units of RC? \( V/A \) \( C/V = C/A = s \)

We call RC the **time constant**: \( \tau = RC \)
Differential equation for \( q(t) \):
\[
\frac{dq(t)}{dt} + \frac{q(t)}{\tau} = \frac{\mathcal{E}}{R}
\]

Solution:
\[
q(t) = \frac{\mathcal{E}}{R} \tau (1 - e^{-t/\tau})
\]

Check:
\[
\frac{dq(t)}{dt} = \frac{\mathcal{E}}{R} e^{-t/\tau}
\]

\( q(t) = \mathcal{E}C(1 - e^{-t/\tau}) \quad i(t) = \frac{\mathcal{E}}{R} e^{-t/\tau} \)
Charging a Capacitor (continued)

\[ q(t) = \mathcal{E}C \left(1 - e^{-t/\tau}\right) \]

\[ q_\infty = \mathcal{E}C \]

\[ q = \mathcal{E}C \left(1 - \frac{1}{e}\right) \]

For \( R=1\,k\Omega \) and \( C=1\,mF \)

\( \rightarrow \tau = RC = 1\,s \)

\[ i(t) = \frac{dq(t)}{dt} = \frac{\mathcal{E}}{R} e^{-\frac{t}{\tau}} \]

\[ i_0 = \frac{\mathcal{E}}{R} \]

\[ i = \frac{1}{e} \frac{\mathcal{E}}{R} \]

Demonstration

as if there was no capacitor!
A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the maximum charge stored on the capacitor?

A. the emf $\mathcal{E}$ of the battery
B. the capacitance $C$ of the capacitor
C. the resistance $R$ of the resistor
D. both $\mathcal{E}$ and $C$
E. all three of $\mathcal{E}$, $C$, and $R$
A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the rate $dq/dt$ at which the capacitor charges?

A. the emf $\mathcal{E}$ of the battery
B. the capacitance $C$ of the capacitor
C. the resistance $R$ of the resistor
D. both $C$ and $R$
E. all three of $\mathcal{E}$, $C$, and $R
How long does it take to charge a capacitor to 99% of full charge?

\[
\mathcal{E}C(1 - 0.01) = \mathcal{E}C\left(1 - e^{-t/\tau}\right)
\]

\[
0.01 = e^{-t/\tau}
\]

\[
\frac{-t}{\tau} = ln0.01 \approx -4.6
\]

\[
t \approx 5RC
\]
Discharging a Capacitor

\[ \mathcal{E} - i(t)R - \frac{q(t)}{C} = 0 \]

To discharge, we just take the battery out (set \( \mathcal{E} = 0 \)):

\[ -i(t)R - \frac{q(t)}{C} = 0 \quad \frac{q(t)}{C} + R \frac{dq(t)}{dt} = 0 \]

\[ q(t) = q_0 e^{-\frac{t}{\tau}} \]

\[ q_0 = V_0 C \]

\( q_0 \) is the initial charge

\( V_0 \) is the initial voltage
Discharging a Capacitor (continued)

\[ q(t) = q_0 e^{-\frac{t}{\tau}} \]

\[ q_0 = V_0 C \]

\[ i(t) = \frac{dq(t)}{dt} = -\frac{V_0}{R} e^{-\frac{t}{\tau}} \]

“-” means that the current flows in the direction opposite to the charging current.
Example

Initially the capacitors are charged to 45V. At \( t = 0 \) the switch is closed. Find \( t_1 \) at which the voltage across the capacitors is reduced to 10V. What is the current at this time?

The potential difference between points \( a \) and \( b \):

\[
V(t) = V(0)e^{-\frac{t}{R \Sigma C \Sigma}}
\]

\[
t_1 = R \Sigma C \Sigma \ln \left[ \frac{V(0)}{V(t_1)} \right] = 80 \Omega \cdot 35 \mu F \cdot \ln 4.5 = 0.0042 \text{s}
\]

\[
i(t_1) = \frac{V(t_1)}{R \Sigma} = \frac{10V}{80 \Omega} = 0.125A
\]
Originally the left-hand switch is closed and the voltage source charges a parallel-plate capacitor to a charge of magnitude \( Q \) on both plates. Then the left-hand switch is opened and the right hand switch is closed so that the capacitor is disconnected from the voltage source \( V \), allowing the capacitor to discharge through a resistor of resistance \( R \). Immediately after the right-hand switch is closed, which of the following is true?

a) The voltage drop across the resistor is \( V \).

b) The current through the resistor is 0.

c) The voltage drop across the capacitor is \( V/2 \).

d) The current through the resistor is \( V/RC \).

e) The voltage drop across the capacitor is 0.
Energy Loss in Charging a Capacitor

How much work does battery do? \( W = \varepsilon Q = \varepsilon^2 C \)

How much energy is stored in the capacitor after charging?

\[
U = \frac{\varepsilon^2 C}{2}
\]

Thus how much energy is dissipated (converted to heat) in the resistor?

By energy conservation: \( W - U = \varepsilon^2 C - \frac{\varepsilon^2 C}{2} = \frac{\varepsilon^2 C}{2} \)  
Half of the work done by the battery is wasted as heat!

Check this explicitly:

\[
i(t) = \frac{\varepsilon}{R} e^{-\frac{t}{\tau}}
\]

\[
\int_0^\infty Ri^2(t)dt = \int_0^\infty \frac{\varepsilon^2}{R} e^{-2t/\tau} dt = \frac{\varepsilon^2}{R} \left( -\frac{\tau}{2} \right) \left( e^{-\infty/\tau} - e^{-0/\tau} \right) = \frac{\varepsilon^2 C}{2}
\]
Which of the following statements about the electric field and electric potential is TRUE?

a) The electric field is the direction in which a charge moves in a vacuum.

b) The electric field is equal to -1 times the voltage gradient.

c) Charged particles are always repelled from regions of high electric potential.

d) Gauss’s law states that the surface integral of the electric potential is directly proportional to the enclosed charge.

e) Electric field lines begin at a negative charge and terminate at a positive charge.