Material: Chapters 21-24

Date/Time: Sunday, October 7, 8:10 to 9:30 PM

Locations:

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Formula Sheet: Single sheet, front and back, hand written.
Recall that for a dielectric-filled capacitor, the capacitance is $K$ times greater than that for a vacuum capacitor:

$$C = K \varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$$

$\varepsilon \equiv K \varepsilon_0$ - permittivity of the dielectric
Energy Stored in a Dielectric-filled Capacitor

What is the work $W$ done when charging the capacitor? This is the energy $U$ that is stored in the capacitor.

$$
\delta W = \delta q_m V \\
V = \frac{q_m}{C}
$$

$q_m$ - charge of mobile electrons in metal
$q_i$ - polarization charge in dielectric

$$
U = W = \frac{1}{C} \int_{q_m=0}^{q_m=Q} q_m \delta q_m = \frac{1}{C} \frac{Q^2}{2} = \frac{1}{2} CV^2
$$

For a given voltage $V$, the energy stored in the dielectric-filled capacitor is **K times greater** than that of the empty (“vacuum-filled”) capacitor.
Energy Stored in a Dielectric-filled Capacitor (cont’d)

**Different experiment:** we charge an “empty” capacitor to charge Q, *disconnect it from the voltage source*, and insert dielectric.

$$U_{filled} / U_{empty} = ?$$

Now $Q$ is fixed (rather than $V$).

Now the final energy is **smaller** than the initial one. How can this be? We have to do negative work in order to counteract the force on dielectric (which is pulled into the capacitor by the field).

Recall: A non-uniform electric field pulls dipoles into the region of greater field:

$$F = qE_1 - qE_2 = qd \frac{dE}{dx}$$
Partially Dielectric-Filled Capacitors

\[ C = C_1 + C_2 = \frac{\varepsilon_0}{d} (A_1 + K_2 A_2) \]

\[ C = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left( \frac{d_1}{\varepsilon_0 A} + \frac{d_2}{\varepsilon_0 k_2 A} \right)^{-1} \]
Initially, only $C_1$ was connected to the voltage source, $C_2$ was uncharged. We disconnect $C_1$ from the source and connect it to $C_2$. Find $V^*$, $Q_1$ and $Q_2$.

After $C_1$ was disconnected from the voltage source, its charge was $Q_1^{\text{init}} = C_1 V$. Connection to $C_2$ results in redistribution of charges, but the net charge ($Q_{\Sigma} = Q_1^{\text{init}}$) is conserved:

$$Q_{\Sigma} = Q_1^{\text{init}} = C_1 V \quad \text{switch in position 1}$$

$$Q_{\Sigma} = C_1 V^* + C_2 V^* \quad \text{switch in position 2}$$

$$V^* = \frac{C_1}{C_1 + C_2} V \quad Q_1 = \frac{C_1}{C_1 + C_2} C_1 V \quad Q_2 = \frac{C_1}{C_1 + C_2} C_2 V$$
A parallel plate capacitor is charged to a total charge $Q$ and the battery removed. A slab of material with dielectric constant $K$ is inserted between the plates. The charge stored in the capacitor

A. Increases  
B. Decreases  
C. Stays the same  
D. Depends on the area and plate separation  
E. Not enough information
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A parallel plate capacitor is connected to a constant voltage source. A slab of material with dielectric constant $K$ is inserted between the plates. The charge stored in the capacitor

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1. Increases
2. Decreases
3. Stays the Same
A parallel plate capacitor is charged to a total charge $Q$ and the battery removed. A slab of material with dielectric constant $K$ is inserted between the plates. The energy stored in the capacitor

$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2$

1. Increases
2. Decreases
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$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2$$
A parallel plate capacitor is charged to a total charge \( Q \) and the battery removed. A slab of material with dielectric constant \( K \) is inserted between the plates. The **force on the dielectric**

1. pulls the dielectric in
2. pushes the dielectric out
3. is zero
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1. pulls the dielectric in
2. pushes the dielectric out
3. is zero

$$U = \frac{1}{2} CV^2 = \frac{1}{2} Q^2$$
We have just seen that the when inserting a dielectric into a capacitor \((Q = \text{const.})\), the electrostatic energy decreases, consistent with the dielectric being pulled into the capacitor.

If, on the other hand, the capacitor is connected to a voltage source \((V = \text{const.})\), the electrostatic energy increases as the dielectric is inserted. What does this mean for the force on the dielectric?

A. The electrostatic energy increases, thus the dielectric must be pushed out of the capacitor by the electrostatic force.
B. We know that the dielectric is pulled into the capacitor, thus the electrostatic energy must actually decrease.
C. There is no net force on the dielectric.
D. The electrostatic energy increases and the dielectric is pulled into the capacitor. Both the energy increase and the energy used to do the pulling are provided by the voltage source.
E. Energy is created in this process. Let’s get a patent, quick!
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E. Energy is created in this process. Let’s get a patent, quick!
Consider two concentric spherical metallic shells, one with radius $R$ and one with radius $2R$. Both have the same charge $Q$. At a point a distance $1.5R$ away from the center, what is the electric field.
6. Two small identical metal balls hold charges of $-10 \mu C$ and $+6 \mu C$, respectively. When they are placed a certain distance apart, the magnitude of the force between them is $F$. They are then allowed to touch and brought back to their original position. The force $F$ between them is now

a) $F/15$ and attractive.
b) $F/15$ and repulsive.
c) $4F/15$ and attractive.
d) $4F/15$ and repulsive.
e) $4F/15$ and repulsive.

\[
\begin{align*}
F &= k \frac{q_1 q_2}{r^2} \\
q_{net} &= q_1 + q_2 = q_1' + q_2' \\
q_1' &= q_2' = \frac{q_{net}}{2} = -2 \mu C
\end{align*}
\]

\[
\frac{F'}{F} = \frac{q_1' q_2'}{q_1 q_2} = \frac{(-2)(-2)}{(-10)(+6)} = -\frac{1}{15}
\]

Charge conservation:

\[
q_{net} = q_1 + q_2 = q_1' + q_2'
\]
14. A solid ball of an insulating material and with radius $R$ has a uniform charge density $\rho$. What is the magnitude of the electric field $E(r)$ at a distance $r < R$ from the center of the ball?

\begin{align*}
\text{enclosed charge} & \quad \frac{q(r' \leq r)}{\varepsilon_0} = \frac{4}{3} \pi r^3 \\
\text{flux} & \quad E(r) = \frac{\rho r}{3\varepsilon_0}
\end{align*}

\[ E(r) = \frac{\rho r}{3\varepsilon_0} \]
Each diagram shows two very large parallel charged sheets with uniform charge densities and separations as shown. Which diagram corresponds to the greatest absolute value of the electric potential difference between the sheets?
Iclicker Question

Each diagram shows two very large parallel charged sheets with uniform charge densities and separations as shown. Which diagram corresponds to the greatest absolute value of the electric potential difference between the sheets?

How would we calculate the electric field/potential difference?
9. Find the equivalent capacitance of the combination.

a) $1/4 \, \mu F$
b) $4 \, \mu F$
c) $4/3 \, \mu F$
d) $3/4 \, \mu F$
e) $3 \, \mu F$
8. The electric potential \( V(x) \) depends on \( x \) in the fashion shown in the first panel (I) below. Which of the figures shown below most precisely describes the electric field \( E_x \) in the \( x \)-direction?

\[
E(x) = -\frac{dV(x)}{dx} \hat{x}
\]
Consider the two concentric spheres shown. The inner sphere is a conductor with total charge $-Q$, inner radius $a$, and outer radius $b$. The outer sphere is a conductor, with total charge $+Q$, inner radius $c$, and outer radius $d$. Taking the potential to be 0 at $r = \infty$, at which of the following radii is the magnitude of the potential largest?

a) $r = a$
b) $r = b$
c) $r = c$
d) $r = d$
e) at both $r = a$ and $r = b$
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c) $r = c$

d) $r = d$

e) at both $r = a$ and $r = b$

\[ E(x) \]

\[ V(x) \]
Good Luck!!