Last lecture review:

Electrostatic potential energy

For two charges $Q$ and $q$: 

$$ U(r) = - \int_{\text{reference point}}^{r} \vec{F}_{el} \cdot d\vec{l} $$

$$ U(r) = - \int_{\infty}^{r} q\vec{E}_Q(r) \cdot d\vec{l} = \frac{qQ}{4\pi\varepsilon_0} \left( \frac{1}{r} \right) $$

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Graphs:

- For ++ or -- charges:
  - $U(r)$ decreases with $r$

- For +- or -+ charges:
  - $U(r)$ increases with $r$
The electrostatic potential energy \( U(x) \) of a charge moving along the \( x \)-axis is given by:

Which plot shows the corresponding electrostatic force \( F(x) \) on the charge?

A. \[ \text{Graph A} \]
B. \[ \text{Graph B} \]
C. \[ \text{Graph C} \]
D. None of them.
The electrostatic potential energy $U(x)$ of a charge moving along the x-axis is given by:

Which plot shows the corresponding electrostatic force $F(x)$ on the charge?

A.  
B.  
C.  
D. None of them.
Superposition: several interacting charges

Just like electrostatic forces and electrostatic fields, electrostatic interaction energies up:

Example: a positive test charge $q_0$ interacting with a (fixed) dipole

$$U_0 = \frac{q_0 q_1}{4\pi \varepsilon_0 \left( \frac{1}{r_{01}} \right)} + \frac{q_0 q_2}{4\pi \varepsilon_0 \left( \frac{1}{r_{02}} \right)} = \frac{q_0 q}{4\pi \varepsilon_0 \left( \frac{1}{r_1} - \frac{1}{r_2} \right)}$$
Charge $q_3 = -q$ (#3) is brought from $\infty$ so that this charge and charges $q_1 = +q$ and $q_2 = -q$ form an equilateral triangle. What is the potential energy of charge $q_3$ in the field of charges $q_1$ and $q_2$?

A. 0  
B. $k \frac{q^2}{d}$  
C. $-k \frac{q^2}{d}$  
D. $k \frac{2q^2}{d}$  
E. $-k \frac{2q^2}{d}$
Charge \( q_3 = -q \) (#3) is brought from \( \infty \) so that this charge and charges \( q_1 = +q \) and \( q_2 = -q \) form an equilateral triangle. What is the potential energy of charge \( q_3 \) in the field of charges \( q_1 \) and \( q_2 \)?

\[
U_3 = \frac{q^2}{4\pi \varepsilon_0} \left( \frac{1}{r_{23}} - \frac{1}{r_{13}} \right)
\]

A. 0
B. \( k \frac{q^2}{d} \)
C. \(-k \frac{q^2}{d} \)
D. \( k \frac{2q^2}{d} \)
E. \(-k \frac{2q^2}{d} \)
Electrostatic Potential

The \textit{electrostatic potential} $V(r)$ at any point $r$ in space is the electrostatic potential energy that a test charge $q$ would have at this point, divided by the test charge $q$:

$$V(r) = \frac{U(r)}{q}$$

Units: J/C=Volt

Potential of a point charge $Q$:

$$V = \frac{1}{4\pi \epsilon_0} \frac{Q}{r}$$

Value of point charge

Distance from point charge to where potential is measured

\textit{Electric potential due to a point charge}
From Electric Potential to Electric Field

\[ F(x) = -\frac{\partial U(x)}{\partial x} \]

\[ E(x) = -\frac{\partial V(x)}{\partial x} \]

\[ F(x) = qE(x) \]

\[ U(x) = qV(x) \]

Potential of a positive charge

\[ V(x, y = \text{const}) \]

\[ E_x(x, y = \text{const}) \]
Calculation of Electrostatic Potential

1. Discrete charges →

\[ V(r) = k \sum_{i} \frac{q_i}{|\vec{r} - \vec{r}_i|} \]

\[ V(\infty) = 0 \]

2. Continuous charge distribution →

\[ V(r) = k \int_{vol} \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau \]

\[ V(\infty) = 0 \]

3. \( \vec{E}(r) \) is known →

\[ V(r) = - \int \limits_{ref.\ point}^{r} \vec{E}(r') \cdot d\vec{l} \]
Iclicker Question

Compare electrostatic potentials at (x) for the following charge distributions:

All charge distributions have the same positive charge density $\lambda$

A. $V_A = V_B = V_C$.

B. $V_A > V_B > V_C$.

C. $V_A > V_B = V_C$.

D. $V_A = V_B > V_C$.

E. none of the others.
Compare electrostatic potentials at (x) for the following charge distributions:

All charge distributions have the same positive charge density $\lambda$

$$V = k \frac{\lambda l_{tot}}{R}$$

A. $V_A = V_B = V_C$.
B. $V_A > V_B > V_C$.
C. $V_A > V_B = V_C$.
D. $V_A = V_B > V_C$.
E. none of the others.
Electric Field and Potential of a Charged Metal Sphere

Though the (continuous) charge distribution is given, it is easier to use our result for the field:

\[
V(r)_{r>R} = - \int_{\text{ref. point}}^{r} \vec{E}(r') \cdot d\vec{l} = - \int_{\infty}^{r} \frac{q}{4\pi \varepsilon_0} \frac{\hat{r}}{r'^2} \cdot dr' \hat{r}
\]

\[
= - \int_{\infty}^{r} \frac{q}{4\pi \varepsilon_0} \frac{dr'}{r'^2} = \frac{q}{4\pi \varepsilon_0} \left( \frac{1}{r} - \frac{1}{\infty} \right) = \frac{q}{4\pi \varepsilon_0 r}
\]

\[
V(r)_{r<R} = \frac{q}{4\pi \varepsilon_0 R}
\]

\[q = q_{\text{net}}\]
Consider two concentric spherical shells, one with radius $R$ and one with radius $2R$. Both have the same charge $Q$. At a point on the outer shell, the magnitude of the electric potential is

A. $2kQ/R$
B. $kQ/R$
C. $kQ/2R$
D. 0
E. $kQ/2R^2$
Consider two concentric spherical shells, one with radius $R$ and one with radius $2R$. Both have the same charge $Q$. At a point on the outer shell, the magnitude of the electric potential is

\[ V = k \frac{Q}{2R} + k \frac{Q}{2R} \]

A. $2kQ/R$

B. $kQ/R$

C. $kQ/2R$

D. 0

E. $kQ/2R^2$
Electric Field and Potential of a Parallel-Plate Capacitor

\[ E(x) \]

\[ V(x) \]
Equipotential lines

- Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.
Equipotential Lines

Equipotential lines (and surfaces) are lines (surfaces) on which the potential (voltage) is constant. They are plotted for fixed differences in voltage.

At each point, a field line is perpendicular to an equipotential line:

\[ \int_{\text{along equipot. line (surface)}} \vec{E}(r) \cdot d\vec{l} = 0 \]
Dipole Field Lines and Equipotential Lines

\[ V(x, y) \]

- \( V = +2V \)
- \( V = +1V \)
- \( V = 0V \)
- \( V = -1V \)
Which point corresponds to the greatest magnitude of the electric field?

A. A  
B. B  
C. C  
D. D

\[ \vec{E}(x, y) = -\frac{\partial V(x, y)}{\partial x} \hat{x} - \frac{\partial V(x, y)}{\partial y} \hat{y} \]
Which point corresponds to the greatest magnitude of the electric field?

A. A
B. B
C. C
D. D

\[ \vec{E}(x, y) = -\frac{\partial V(x, y)}{\partial x} \hat{x} - \frac{\partial V(x, y)}{\partial y} \hat{y} \]