Last lecture: Gauss’ Law:

\[ \oint \vec{E}(\vec{r}) \cdot d\vec{A} = \frac{Q_{\text{net}}}{\varepsilon_0} \]
Consider a closed Gaussian surface. Which of the following is false?

A. If there is no net charge enclosed by the surface, then the total electric flux through the surface is zero.

B. If the electric field is zero everywhere on the surface, then there can be no net charge enclosed by the surface.

C. If the total electric flux through the surface is zero, then the total charge enclosed by the surface is zero.

D. If the electric field is zero everywhere on the surface, then the total electric flux through the surface is zero.

E. If the total electric flux through the surface is zero, then the electric field must be zero everywhere on the surface.
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C. If the total electric flux through the surface is zero, then the total charge enclosed by the surface is zero.

D. If the electric field is zero everywhere on the surface, then the total electric flux through the surface is zero.

E. If the total electric flux through the surface is zero, then the electric field must be zero everywhere on the surface.
An electric field in a certain region has constant magnitude and direction. From this information alone, we can conclude that

A. there must be positive charge distributed throughout that region
B. there must be negative charge distributed throughout that region
C. there must be zero net charge at all points within that region
D. such an electric field is physically impossible
E. nothing useful about the presence or absence of charge in that region.
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D. such an electric field is physically impossible

E. nothing useful about the presence or absence of charge in that region.
**Example:** electric field of a uniformly charged ball. Radius $R$, total charge $Q$.

Gaussian surface: a sphere centered at ($\cdot$) $O$. Its radius $r$ can be smaller or larger than $R$.

The field outside the sphere is the same as that for a point charge $Q$ at the sphere’s center.
**Gauss’ Law: Infinite Cylinder**

**Example:** the field of uniformly charged infinitely long wire.

Gaussian surface: a cylinder of length $L$ centered at the wire.

$E(r) \cdot 2\pi rl = \frac{\rho l}{\varepsilon_0}$

$E(r) = \frac{\rho}{2\pi \varepsilon_0 r}$

$\vec{E}(r) = E(r)\hat{r}$

- $\rho$ - the linear density of charges (the unit length charge)

$r \geq R$
Gauss’ Law: Plane Symmetry

**Example:** the field of uniformly charged infinite plane.

Gaussian surface: a rectangular box (or cylinder) centered at the plane.

\[
\mathbf{E}(x) \cdot 2A = \frac{\sigma A}{\varepsilon_0}
\]

enclosed charge

\[
\mathbf{E}(x) = \frac{\sigma}{2\varepsilon_0}
\]

\[
\mathbf{E}(x) = \pm \mathbf{E}(x)\hat{x}
\]

\[
\sigma - the surface charge density \quad [\text{C/m}^2]
\]
Superposition

\[ E_{\text{net}} = E_1 + E_2 \]

\[ E(x) \]

\[ x \]

\[ E_1 \quad E_2 \]

\[ E_{\text{net}} = E_1 + E_2 \]
A charge \( q \) is at the center of a cube. What’s the flux of \( E \) through the shaded side?

\[
\Phi_E = \frac{q}{\varepsilon_0}
\]

(A) \( \Phi_E = \frac{q}{\varepsilon_0} \)

(B) \( \Phi_E = \frac{q}{\varepsilon_0} \times \frac{1}{2} \)

(C) \( \Phi_E = \frac{q}{\varepsilon_0} \times \frac{1}{3} \)

(D) \( \Phi_E = \frac{q}{\varepsilon_0} \times \frac{1}{6} \)

(E) \( \Phi_E = \frac{q}{\varepsilon_0} \times \frac{1}{12} \)
A charge $q$ is at the center of a cube. What’s the flux of $E$ through the shaded side?

(A) $\Phi_E = \frac{q}{\epsilon_0}$

(B) $\Phi_E = \frac{q}{\epsilon_0} \times \frac{1}{2}$

(C) $\Phi_E = \frac{q}{\epsilon_0} \times \frac{1}{3}$

(D) $\Phi_E = \frac{q}{\epsilon_0} \times \frac{1}{6}$

(E) $\Phi_E = \frac{q}{\epsilon_0} \times \frac{1}{12}$
Matter in Electrostatic Field

**Conductors:**

- Mobile electrons

\[
\text{Electric field} \rightarrow \text{current}
\]

**Insulators (Dielectrics):**

- Induced/built-in dipoles

\[
\text{Electric field} \rightarrow \text{polarization}
\]
What happens if you place a piece of metal in an electric field?

Q: Can there be an electric field inside a metal?
A: Yes, but only if there is a current flowing!

- In electrostatics, we consider only “static” charge distributions, i.e., the charges are not moving.
- If a piece of metal is placed in an electric field, the electrons will be redistributed (transient currents flow) until the field is zero everywhere inside the metal, at which time the forces on the electrons are zero, and currents stop flowing.

→ $E = 0$ inside a conductor (in electrostatics)
Conducting Sphere in the Field of a Point Charge

In the final (static) state, inside the metal the field of the external charge is exactly canceled by the field of induced surface charges!

Why?
If the field wasn’t canceled, there would still be forces acting on the conduction electrons, leading to further charge redistribution!

Field of point charge + Field of induced surface charges

Net field $E = 0$ inside the metal
What is the surface charge density?

Use Gauss’ law!

Inside the metal: field free

Outside the metal: $E = \frac{\sigma}{\varepsilon_0}$

Gaussian surface

$A \ E = \frac{Q}{\varepsilon_0}$

$A \ E = A \ \frac{\sigma}{\varepsilon_0}$

$E = \frac{\sigma}{\varepsilon_0}$
At metallic surfaces the electric field is directed **along the normal vector to the surface**.

The surface charge density (and, thus, the field intensity) is higher near protrusions: the electrons arrange themselves in the way that minimizes the total energy of this charge distribution.
A conducting shell/cage can completely shield an external electric field:

Field pushes electrons toward left side. Net positive charge remains on right side.

Field perpendicular to conductor surface

\[ \vec{E} = 0 \]

This is why you are (relatively) safe from lightening strike inside a car!
Charges within Cavities

Using Gauss’ law, we can easily find the net surface charge if we place a charge inside a metallic shell:

\[ Q_{\text{in}} = -Q \]

\[ Q_{\text{out}} = +Q \]

\[ E=0 \text{ inside metal} \]

\[ \Phi_E = 0 \quad \text{for any Gaussian surface inside metal} \]

\[ Q_{\text{encl}} = 0 = +Q + Q_{\text{in}} \quad Q_{\text{in}} = -Q \]

\[ Q_{\text{metal}} = 0 = Q_{\text{in}} + Q_{\text{out}} \quad Q_{\text{out}} = -Q_{\text{in}} = +Q \]
Consider a conducting sphere, with a cavity inside. We insert a 5 μC charge into the cavity. What is the charge on the inner and outer surfaces of the conductor?

A. 0 μC, 0 μC.
B. -5 μC, -5 μC.
C. -5 μC, 5 μC.
D. 5 μC, -5 μC.
E. 5 μC, 5 μC.
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C. -5 μC, 5 μC.
D. 5 μC, -5 μC.
E. 5 μC, 5 μC.
**Example: Parallel-Plate Capacitor**

**Parallel-plate capacitor:**
(two metallic plates with charges $+q$ and $-q$)

$S_1$: charge density $\sigma$, inside $E=0$, outside $\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{x}$

$S_2$: charge density 0, inside $E=0$, outside $E=0$

$S_3$: charge density 0, inside $E=0$, outside $E=0$

$S_4$: charge density $-\sigma$, inside $E=0$, outside $\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{x}$
Example: Concentric Conducting Spherical Shells

Charge densities:

\[
\begin{align*}
\sigma(r = a) &= 0 \\
\sigma(r = b) &= \frac{+Q}{4\pi b^2} \\
\sigma(r = c) &= \frac{-Q}{4\pi c^2} \\
\sigma(r = d) &= 0
\end{align*}
\]

Electric fields:

\[
\begin{align*}
E(r < b) &= 0 \\
E(b < r < c) &= \frac{Q}{4\pi \varepsilon_0 r^2} \\
E(r > c) &= 0
\end{align*}
\]

Inner shell: +Q

Outer shell: -Q

\[
E(r = b^+) = \left\{ \begin{array}{l}
\frac{\sigma(r = b)}{\varepsilon_0} = \frac{Q}{4\pi \varepsilon_0 b^2} \\
\left. \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \right|_{r=b} = \frac{Q}{4\pi \varepsilon_0 b^2}
\end{array} \right.
\]