Welcome to 227 “ELECTROMAGNETISM”

The course Web site: http://www.physics.rutgers.edu/ugrad/227fall/

Lecturers: Prof. Frank Zimmermann

Course administrator: Prof. Roy Montalvo
• Lectures are mandatory
• Iclickers 4% of grade (register your iclicker, using RUID not NetID)
• Reading assignments (reading quizzes?)
Enormous range of electromagnetic phenomena (and devices based on them):

All of them (except interactions of the electromagnetic field with matter) can be described by 4 (!) equations (Maxwell Eqs).

Fundamental goal of the course: to appreciate this unity.
Electric Charge

Electric charge is a physical quantity that characterizes how charged objects participate in electrostatic interactions (compare: mass – the charge for gravitational interactions).

Electric charge properties:

- It’s a scalar (remains the same in all ref. frames). Charge comes in two types — positive and negative.

- It’s quantized – it’s allowed to be integer multiples of $\pm e$ (the elementary charge).

\[ e \approx 1.6 \times 10^{-19} \text{ C} \]

Unit of charge: Coulomb

Oil drop experiment 1909

Harvey Fletcher (1884 - 1981)

Robert A. Millikan (1868 - 1953)

Nobel 1923

- The net charge of any isolated system is conserved (this, of course, doesn’t mean that the number of particles is conserved). The net charge of the Universe is believed to be 0. Thus, its large-scale structure is shaped by gravity, not by electromagnetic forces (which are much stronger).
Coulomb’s Law

The force of interaction between two **point** charges **at rest in vacuum** is directed along the line connecting the charges, it is proportional to the charges $q_1$ and $q_2$ and inversely proportional to the (square of the) distance between the charges.

\[
F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2}
\]

\[
F(\text{gravity}) = G \frac{|m_1 m_2|}{r^2}
\]

\[
\varepsilon_0 \approx 9 \times 10^{-12} C^2/(Nm^2)
\]

\[
\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9 Nm^2/C^2 \approx 10^{10} Nm^2/C^2
\]
Coulomb's Law

\[ \mathbf{F}_{q-Q} = \mathbf{F}_{Q-q} \]

What is the x-component of \( \mathbf{F}_{q-Q} \) (the force that \( q \) exerts on \( Q \))?
What is the $x$-component of $\vec{F}_{q-Q}$ (the force that $q$ exerts on $Q$)?
How strong?

Electromagnetic interaction is billion-...- billion times stronger than the gravitational one. Why do we usually ignore it? Because the matter around us is mostly uncharged (almost perfect balance of positive and negative charges).

To appreciate the strength of electric forces, let's calculate the force of repulsion between two tough guys with $10^{-9}$ (one part per billion) of their electrons removed:

charge (10^{-9} of all electrons removed): $q \approx 5C$

$$\begin{align*}
\text{# of H}_2\text{O molecules: } & \text{ Avogadro } \times \# \text{ moles } = 6 \cdot 10^{23} \text{ molecules } \frac{100 \text{ kg}}{18 \text{ g mol}} = 3 \cdot 10^{27} \\
\text{# of electrons: } & \# \text{ of H}_2\text{O molecules } \times 10 = 3 \cdot 10^{28} \quad q = 3 \cdot 10^{19} \times 1.6 \cdot 10^{-19} C \approx 5C \\
\text{force: } F &= \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r^2} \approx 10^{10} N m^2/C^2 \frac{5C \times 5C}{(0.5m)^2} \approx 10^{12} N
\end{align*}$$

A force strong enough to lift 1000 aircraft carriers (100,000 tons each)!
**Superposition Principle:** Interaction between any two charges is completely unaffected by the presence of the other charges.

\[
\vec{F}_{\text{net}} = \sum_i \vec{F}_i
\]

\[
\vec{F}_{\text{net}} = \vec{F}_{2-3} + \vec{F}_{1-3}
\]

\[
\approx 0.005N \cdot \hat{x} - 0.005N \cdot \hat{y}
\]

\(\hat{x}\) - the unit vector along \(\hat{x}\)

\(\hat{y}\) - the unit vector along \(\hat{y}\)

\[
F = \sqrt{F_x^2 + F_y^2} \approx 0.007N
\]
Maxwell Equations are *linear* differential equations; they have solutions which can be added together to form other solutions. This is a reflection of the fact that photons *do not* interact with each other.

**Foundations of Electrostatics: Coulomb’s Law and Superposition Principle.**

The Main Problem of Electrostatics:

Given: the charge distribution (charges at rest)

Find: the force on a probe charge at any point in space
Coulomb's law in coordinate form

\[ F = |\vec{F}_{2-1}| = |\vec{F}_{1-2}| = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Coulomb's Law in vector form

\[ \vec{F}_{1\to 2} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{(\vec{r}_2 - \vec{r}_1)^2} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \]

force on \( q_2 \) by \( q_1 \)

unit vector along \( \vec{r}_2 - \vec{r}_1 \)

If one of the charges is at the origin \( (r_1=0) \):

\[ \vec{F}_{1\to 2} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{(\vec{r}_2)^2} \frac{\vec{r}_2}{|\vec{r}_2|} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{(\vec{r})^2} \frac{\vec{r}}{|\vec{r}|} \]

\[ \vec{F}_{2\to 1} = ? \]
Three point charges lie on the x-axis. All three charges have the same magnitude \(+1\text{C}\). The positions of the charges are shown on the plot. Find the force exerted on \(\#3\) by \(\#1\) and \(\#2\).

\[
\begin{align*}
\#1 & \quad \#2 & \quad \#3 \\
-1\text{m} & \quad 0\text{m} & \quad 1\text{m}
\end{align*}
\]

A. \( F = \frac{1}{4\pi\varepsilon_0} \frac{1}{2} \frac{C^2}{m^2} \)  
B. \( F = \frac{1}{4\pi\varepsilon_0} 2 \frac{C^2}{m^2} \)  
C. \( F = \frac{1}{4\pi\varepsilon_0} \frac{1}{3} \frac{C^2}{m^2} \)  
D. \( F = \frac{1}{4\pi\varepsilon_0} \frac{5}{4} \frac{C^2}{m^2} \)  
E. \( F = \frac{1}{4\pi\varepsilon_0} \frac{3}{2} \frac{C^2}{m^2} \)
Three point charges lie on the x-axis. All three charges have the same magnitude \( +1 \text{C} \). The positions of the charges are shown on the plot. Find the force exerted on \( #3 \) by \( #1 \) and \( #2 \).

\[
\begin{align*}
#1 & \quad #2 & \quad #3 \\
-1 \text{m} & \quad 0 \text{m} & \quad 1 \text{m}
\end{align*}
\]

**Iclicker Question**

A. \( F = \frac{1}{4\pi \varepsilon_0} \frac{1}{2} \frac{C^2}{m^2} \)  
B. \( F = \frac{1}{4\pi \varepsilon_0} 2 \frac{C^2}{m^2} \)

C. \( F = \frac{1}{4\pi \varepsilon_0} \frac{1}{3} \frac{C^2}{m^2} \)  
D. \( F = \frac{1}{4\pi \varepsilon_0} \frac{5}{4} \frac{C^2}{m^2} \)  
E. \( F = \frac{1}{4\pi \varepsilon_0} \frac{3}{2} \frac{C^2}{m^2} \)
An electric dipole is formed by two charges $+q$ and $-q$ at the distance $d$ apart. Find the force on a probe charge $+q_p$ which is placed at a distance $r$ from the middle of the dipole along the line perpendicular to the dipole axis.

**Step 1:** nice sketch and notations

**Step 2:** Given: $q$, $q_p$, $d$, $r$. Find: $F_{\text{net}}$

**Step 3:** analyze the problem

**Concepts:** Coulomb’s law and superposition principle.

**Symmetry:** horizontal components of $F_1$ and $F_2$ compensate each other, we need to consider only vertical components.

\[
F_{+y} = F_+ \sin \alpha = \frac{1}{4\pi \varepsilon_0} \frac{q_p q}{r^2 + (d/2)^2} \sin \alpha = \frac{1}{4\pi \varepsilon_0} \frac{q_p q}{r^2 + (d/2)^2} \sqrt{r^2 + (d/2)^2}
\]

\[
F_{\text{net}} = F_{+y} + F_{-y} = \frac{1}{4\pi \varepsilon_0} \frac{q_p q d}{[r^2 + (d/2)^2]^{3/2}}
\]
Electric dipole (cont’d)

\[ F_{\text{net}} = \frac{1}{4\pi \varepsilon_0} \left( \frac{q_p q d}{r^2 + (d/2)^2} \right)^{3/2} \to (r \gg d) \approx \frac{1}{4\pi \varepsilon_0} \frac{q_p q d}{r^3} \]

– at \( r \gg d \), the force decreases with distance faster than that for a point charge. This makes sense because the net charge of the dipole is 0.


Iclicker Question

Correct representation of \( F_y(x) \):

A

B

C

D

E
Electric dipole (cont’d)

\[ F_{\text{net}} = \frac{1}{4\pi\varepsilon_0} \frac{q_p q d}{[r^2 + (d/2)^2]^{3/2}} \quad (r \gg d) \approx \frac{1}{4\pi\varepsilon_0} \frac{q_p q d}{r^3} \]

- at \( r \gg d \), the force decreases with distance faster than that for a point charge. This makes sense because the net charge of the dipole is 0.


**Iclicherk Question**

Correct representation of \( F_y(x, y = 0) \):
Outline of Course

Electromagnetism

$E$ - electric fields

$B$ – magnetic fields

$Q$ – charges

$I$ – currents

$\Phi$ - fluxes

$d \frac{dt}{dt} = 0$

$d \frac{dt}{dt} \neq 0$

Electric fields generated by charges at rest

Ch. 21 - 26

Magnetic fields generated by time-independent currents

Ch. 27 - 28

Electro-Magneto statics

El.-mag. fields generated by alternating currents

Ch. 29 - 31

Electromagnetic Waves

Ch. 32

El.-mag. fields generated by alternating currents
All of them (except interactions of the electromagnetic field with matter) can be described using Maxwell Equations.