The first common hour midterm exam will be held on Thursday October 1, 9:50 to 11:10 PM (at night) on the Busch campus. You should go to the room corresponding to the first 3 letters of your last name. If you have a conflict with the exam time, please contact Prof. Cizewski Cizewski@physics.rutgers.edu with your entire schedule for the week of September 28 at your earliest convenience but not later than 5:00 pm on Wednesday, September 23.

Aaa – Hoz  ARC 103  
Hua – Moz  Hill 114  
Mua – Shz  PHY LH  
Sia – Zzz  SEC 111
If you have problems with your iClicker registration: Email Professor Gershenson (gersh@physics.rutgers.edu) with your name and iClicker number for help!
Electrostatic Potential

**Electrostatic potential**

\[ V(r) = \frac{U(r)}{q} \]

Units: J/C=Volt

Potential is related (but not equal!) to the potential energy of charges in the electric field. Numerically, this is the work done by *external* forces ("us") to bring a positive unit charge from some reference point to the point in question.
From El. Potential to El. Field

\[
\vec{F}(x, y) = -\frac{\partial U(x, y)}{\partial x} \hat{x} - \frac{\partial U(x, y)}{\partial y} \hat{y}
\]

\[
\vec{E}(x, y) = -\frac{\partial V(x, y)}{\partial x} \hat{x} - \frac{\partial V(x, y)}{\partial y} \hat{y}
\]

\[
\vec{F}(x, y) = q\vec{E}(x, y)
\]

\[
U(x, y) = qV(x, y)
\]

Potential of a positive charge
Calculation of Electrostatic Potential

1. Discrete charges →

\[
V(r) = k \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|}
\]

\[V(\infty) = 0\]

2. Continuous charge distribution →

\[
V(r) = k \int_{vol} \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau
\]

\[V(\infty) = 0\]

3. \(\vec{E}(r)\) is known →

\[
V(r) = - \int_{\text{ref.point}}^r \vec{E}(r) \cdot d\vec{l}
\]
Example: Electric Field and Potential of a Charged Ring

Find $V(x)$ along the axis of symmetry.

\[ V(r) = -\int_{\text{ref. point}}^r \vec{E}(r) \cdot d\vec{l} \]

\[ V(r) = k \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|} \]

\[ V(r) = k \int_{\text{vol}} \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau \]

\[ V(x) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{\sqrt{x^2 + a^2}} \]

\[ V(x) = \frac{1}{4\pi \varepsilon_0} \int \frac{\lambda(\vec{r})}{|\vec{r} - \vec{r}'|} d\vec{l} = \frac{1}{4\pi \varepsilon_0} \frac{\Phi \lambda dl}{\sqrt{x^2 + a^2}} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{\sqrt{x^2 + a^2}} \]

\[ V(x) = -\frac{Q}{4\pi \varepsilon_0} \int_{\infty}^{x_0} \frac{1}{x^2 + a^2} \frac{xdx}{\sqrt{x^2 + a^2}} = (y = x^2 + a^2, dy = 2xdx) = -\frac{Q}{4\pi \varepsilon_0} \frac{1}{2} \int_{\infty}^{x_0} \frac{dy}{y^{3/2}} \]

\[ \propto \frac{1}{x} \quad (x \gg a) \]

\[ E_x(x) \]

\[ x \]

\[ y = x^2 + a^2, \quad dy = 2xdx \]

\[ \int \frac{dy}{y^{3/2}} \]

\[ 10 \]
Electric Field and Potential of a Charged Metal Sphere

Though the (continuous) charge distribution is given, it is easier to use our result for the field:

\[
V(r)_r>R = - \int_{r}^{\infty} \vec{E}(r) \cdot d\vec{l} = - \int_{r}^{\infty} \frac{q}{4\pi\varepsilon_0 r^2} \cdot d\vec{r} = - \int_{r}^{\infty} \frac{q}{4\pi\varepsilon_0 r^2} \cdot d\vec{r} = \frac{q}{4\pi\varepsilon_0 r}
\]

\[
V(r)_r<R = ? \quad V(r)_r<R = \frac{q}{4\pi\varepsilon_0 R}
\]
Electric Field and Potential of a Parallel-Plate Capacitor

Potential is a continuous function of $r$. 
(exception: point charges).
Equipotential lines (and surfaces) are lines (surfaces) on which the potential (voltage) is constant. They are plotted for fixed differences in voltage.

At each point, a field line is perpendicular to an equipotential line:

\[ \int_{\text{along equipot. line (surface)}} \mathbf{E}(r) \cdot d\mathbf{l} = 0 \]
Example: Dipole

\[ V(x, y) \]

\[ V = +2V \]
\[ V = +1V \]
\[ V = 0V \]
\[ V = -1V \]
If an equipotential line crosses itself, then $E=0$ at this point.

Recall: a field line never crosses itself!
**Dangers of the Web**

*Compare these two plots. What’s wrong with the right one?*

The electric field intensity is proportional to the density of equipotential lines.

At the center \( \vec{E}(x, y, z) = - \frac{\partial V(x, y, z)}{\partial x} \hat{x} \).

Touching equipotential lines (corresponding to different potentials) imply an infinitely strong field.
Electrostatic potential energy and electrostatic potential.
Connection between $V(r)$ and $E(r)$
Equipotential surfaces and field lines.

Next time: Lecture 7. Capacitors and Electric Field Energy
§§ 24.1, 24.3
Four equal charges as a square

“there are five equilibrium points (the green circles); only the one in the center of the square is stable. A test charge placed near this point would invariably end up in the equilibrium position, which is a local minimum for the potential. The other four points are local maxima of the potentials”.

What’s wrong with these field lines/potential surfaces?

*It’s all wrong*: the field lines cross at the center (where no charges exist), and the potential forms a local minimum in the space free of charges.
Appendix II. Earnshaw’s Theorem

The plots show equipotential lines that correspond to configurations of point charges. Which plot is incorrect?

**Earnshaw's theorem**: A charged body cannot be held in stable stationary equilibrium by electrostatic forces from other charged bodies.

For a system governed by electrostatics there can be no potential energy minimum, or maximum, in an unoccupied by charges region of space.

At a local energy minimum a positively charged particle would have to feel a restoring force no matter which way it is displaced. This could only be at a point where lines of force all converge. Likewise, the only stable position for a negative charge would be where lines of force all diverge. But each of these conditions is true only for a point where **there is a particle of the opposite charge**. By the same token, the only local energy maximum for a particle is where there is another particle of the same charge.

The Earnshaw's Theorem explains why you are not accustomed to seeing things levitating by electrostatic forces. For the same reason, we cannot use electrostatic forces for plasma confinement in thermonuclear power plants.