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Office hours: Thursdays 3 – 4 PM (may be changed)
The first common hour midterm exam will be held on **Thursday October 1, 9:50 to 11:10 PM** (at night) on the Busch campus. You should go to the room corresponding to the first 3 letters of your last name. If you have a conflict with the exam time, please contact Prof. Cizewski Cizewski@physics.rutgers.edu with your entire schedule for the week of September 28 **at your earliest convenience but not later than 5:00 pm on Wednesday, September 23.**

Aaa – Hoz       ARC 103  
Hua – Moz       Hill 114  
Mua – Shz       PHY LH  
Sia – Zzz       SEC 111
General problem of Electrostatics:

**Given:** objects that carry charges (point-like, extended, different materials)

**Find:** the electric field everywhere.

To solve the problem, we need to combine

the field equation (Coulomb’s Law = Gauss’ Law) + the “materials” equation (macroscopic description of the matter’s response to the electric field)

The simplest materials equation corresponds to the case of metals (conductors) in the electrostatic field.
Reminder: Gauss’s law

\[ \int \vec{E}(\vec{r}) \cdot d\vec{A} = \frac{Q_{\text{net}}}{\varepsilon_0} \]
Matter in Electrostatic Field

Conductors

Mobile electrons

Electric field → current

Dielectrics

Induced/built-in dipoles

Electric field → polarization

Though the mechanisms of polarization in metals and dielectrics are different, we can treat metals (for now) as an extreme case of highly-polarizable dielectrics.
**Metals**: huge number of free-moving electrons (conducting electrons).

\[ E=0 \text{ inside a metal} \quad - \text{the “materials” equation} \]

If a piece of metal is placed in electric field, the electrons will be redistributed (current flows) until the field is 0 everywhere inside inside the metal.

The equilibrium charge distribution: no net charge in the volume, the surface is charged in such a way that the field of this surface charge compensates the external field in the volume (the electric field lines are terminated and originated at the surface).
Neutral Conducting Sphere in the Field of a Point Charge

Field of Point Charge

Field of Surface Charges

Ignore the cavity!

Net Field
\((E=0 \text{ inside metal})\)

http://web.mit.edu/viz/EM/visualizations/electrostatics/ChargingByInduction/shielding/shielding.htm
Surface Charge Density $\sigma$

External field (without a metal)

\[ E_{S} = \frac{\sigma}{2\varepsilon_{0}} \]

Field of surface charges

Superposition:

\[ E_{net} = E + \frac{\sigma}{2\varepsilon_{0}} = \frac{\sigma}{\varepsilon_{0}} \]

outside: $E_{net} = E + \frac{\sigma}{2\varepsilon_{0}} = \frac{\sigma}{\varepsilon_{0}}$

inside: $E_{net} = E - \frac{\sigma}{2\varepsilon_{0}} = 0$

\[ E_{net} = 2E \]

\[ \sigma = 2\varepsilon_{0}E \]
Comparison: charged metallic and dielectric spheres

Uniformly charged (dielectric!) sphere

Charged conducting sphere

The same field outside if the net Q is the same.
At metallic surfaces the electric field is directed along the normal vector to the surface.

The surface charge density (and, thus, the field intensity) is higher near protrusions: the electrons arrange themselves in the way that minimizes the total energy of this charge distribution.
3 atoms as seen by the Rutgers Helium Ion Microscope
Neutral Conducting Shell in the Field of a Point Charge

Field of Point Charge + Field of Surface Charges

Net Field:
\[ E = 0 \text{ everywhere inside} \]

http://web.mit.edu/viz/EM/visualizations/electrostatics/ChargingByInduction/shielding/shielding.htm
Faraday Cage (Shield)

Man-made “lightning”...

.. and a real one

Faraday cage in my lab
The first midterm!!

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Cavities (no Charges within the Cavity)

This conclusion holds for any inner surface: the charge density on the inner surface is zero provided no charges within the cavity.

\[ Q_{net} = +Q = Q_{in} + Q_{out} \]

\[ \sigma(r = a) = 0 \]

\[ \sigma(r = b) = \frac{+Q}{4\pi b^2} \]

Shell charge: +Q

\( E \) field lines
In general, it is difficult to calculate the charge distribution on the surface of a conductor placed in an external electric field (see the Figure).

We can easily find the net surface charge if we place a charge inside a metallic shell.

Our logic:

\[ E = 0 \text{ inside metal} \]

\[ \Phi_E = 0 \text{ for any Gaussian surface inside metal} \]

\[ Q_{encl} = 0 = +Q + Q_{in} \quad Q_{in} = -Q \]

\[ Q_{metal} = 0 = Q_{in} + Q_{out} \quad Q_{out} = -Q_{in} = +Q \]
\( \sigma_{\text{out}} \) does not depend on the \textit{position} of \( Q \) inside - for a spherical shell \( \sigma_{\text{out}} \) must be uniform (this minimizes the energy of this system of charges).

The field outside is completely insensitive to the position/motion of charge \( Q \) inside!

Thus, the metal shell divides the whole space into two regions:
- “Outside” knows about the total charge enclosed, but cannot detect the charge position
- “Inside” knows nothing about the magnitude and positions of the outside charges.
Problem 22.45: Concentric Spherical Shells

Charge densities:
\[ \sigma(r = a) = 0 \]
\[ \sigma(r = b) = \frac{+Q}{4\pi b^2} \]
\[ \sigma(r = c) = \frac{-Q}{4\pi c^2} \]
\[ \sigma(r = d) = 0 \]

Inner shell: +Q
Outer shell: -Q

Electric fields:
\[ E(r < b) = 0 \]
\[ E(b < r < c) = \frac{Q}{4\pi \varepsilon_0 r^2} \]
\[ E(r > c) = 0 \]
Conclusion

Metals in electrostatic fields: \( E = 0 \) inside.

Next time: Lecture 5. Electric Potential
\( \S \S \) 23.1-23.4
Field calculations based on the integral form of GL are not limited to these trivial cases: any charge distribution that can be considered as \textit{superposition} of symmetrical charge distributions can be treated on the basis of this law.

1. Two infinite planes with the charge densities $\sigma$ and $2\sigma$. 

$$E = \frac{\sigma \sqrt{1 + 4}}{2\varepsilon_0}$$

2. Uniformly charged spherical shell with a small hole in it. Find $E$ in the hole.

Hole can be considered as (almost flat) “patch” of negative charge density ($-\sigma$) on top of a positively charged ($\sigma$) whole sphere.

Field of the sphere 

$$E_S = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0}$$

$E_S = 0$

Field of the patch 

$$E_p = \frac{\sigma}{2\varepsilon_0}$$

Superposition! 

$$E_{net} = \frac{\sigma}{2\varepsilon_0}$$
Appendix II: Electric Field of a Charged Sphere with a Spherical Cavity

Consider a uniformly charged sphere with a spherical cavity. Calculate the electric field intensity along the dashed line.

**Hint**: treat cavity as a superposition of positively and negatively charged spheres.

\[
E(x)_{x>R} = \frac{Q}{4\pi \varepsilon_0 x^2} - \frac{Q/8}{4\pi \varepsilon_0 (x - R/2)^2}
\]
Appendix III. Conductors (Metals) in Electrostatics

**Metals**: huge number of free-moving electrons (conducting electrons), \( \sim 10^{28} \) electrons/m³.

Let’s consider an uncharged piece of metal \( (q_{\text{net}}=0) \), and place this metal in an external electric field. The mobile electrons (negative charges) will move with respect to the positively charged ions (polarization)! The equilibrium charge distribution: no net charge in the volume, the surface is charged in such a way that the field of this surface charge compensates the external field in the volume (the electric field lines are terminated and originated at the surface).

High density of mobile charges ensures that any external electric field, no matter how strong, will be «screened» by a very thin surface charge sheet:

\[
\sigma = E \varepsilon_0 = 10^6 \frac{V}{m} \times 9 \cdot 10^{-12} \frac{C}{N \cdot m^2} \approx 10^{-5} \frac{C}{m^2} = 10 \frac{\mu C}{m^2}
\]

\[
t = \frac{\sigma}{e \cdot \text{concentr.}} = \frac{10^{-5} \frac{C}{m^2}}{1.6 \cdot 10^{-19} \frac{C}{1 \cdot 10^{28} m^3}} \approx 10^{-14} m - \text{the size of a nucleus!}
\]
**Appendix IV. Parallel-Plate Capacitor**

**Parallel-plate capacitor:**
(two metallic plates with charges $+q$ and $-q$)

- $S_1$: charge density $\sigma$, inside $E=0$, outside $\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{x}$
- $S_2$: charge density 0, inside $E=0$, outside $E=0$
- $S_3$: charge density 0, inside $E=0$, outside $E=0$
- $S_4$: charge density $-\sigma$, inside $E=0$, outside $\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{x}$

![Diagram of a parallel-plate capacitor with Gaussian surfaces](image)