Attention: the list of unregistered iclickers will be posted on our Web page after this lecture.

From the concept of electric field flux – to the calculation of electric fields of complex charge distributions.
Charged particles are fixed on grids having the same spacing. Each charge has the same magnitude $Q$ with signs given in the figure. Rank the magnitude of the electric field (from strong to weak) at the location marked with an “x”.

A. $1 > 2 > 3 > 4$
B. $2 > 3 > 1 = 4$
C. $1 = 4 > 3 > 2$
D. $3 > 2 > 1 > 4$
A negative point charge $-Q$ is released from rest in an electric field. At this moment, the acceleration of the point charge

A. is in the direction of the electric field at the position of the point charge.

B. is directly opposite the direction of the electric field at the position of the point charge.

C. is perpendicular to the direction of the electric field at the position of the point charge.

D. is zero.

E. not enough information given to decide
A positive point charge $+Q$ is released from rest in an electric field. At any later time, the velocity of the point charge

A. is in the direction of the electric field at the position of the point charge.
B. is directly opposite the direction of the electric field at the position of the point charge.
C. is perpendicular to the direction of the electric field at the position of the point charge.
D. is zero.
E. not enough information given to decide
(Laminar) flow of fluid: the speed of water, \( v \), and the pipe cross section, \( A \), are known. Calculate how much water you added to the bucket in 1 sec.

\[
\Delta V = A \cdot v \cdot 1s
\]

length (water displacement in 1s)

\[
\Delta V = (\vec{A} \cdot \vec{v}) \cdot 1s
\]

scalar product, \( A \cdot v \cdot \cos \phi \)
The concept of flux applies to any vector field $\vec{a}(\vec{r})$ (the field of velocities in a fluid flow, the $E$ field, etc.).

The flux of the field $\vec{a}(\vec{r})$ through a small element of surface $d\vec{A}$:

$$\Phi_a \equiv \vec{a}(\vec{r}) \cdot d\vec{A} = a(r)A \cdot \cos\phi$$

Scalar!
Units of Flux

The flux of electrostatic field:

\[ \Phi_E \rightarrow \frac{N}{C} m^2 \]

\[ (E \rightarrow \frac{N}{C}) \]

The flux of gravitational field:

\[ \Phi_g \rightarrow \frac{N}{kg} m^2 = \frac{m^3}{s^2} \]

\[ (g \rightarrow \frac{N}{kg} = \frac{m}{s^2}) \]
Calculate the flux of uniform electric field through the surface shown in the Figure.

\[ \Phi_E = E(r) \cdot A \cdot \cos \phi \]

- \[ \phi = 120^0 \]
- \[ \cos(120^0) = -0.5 \]

\[ E = 2 \text{ N/C} \]

**A.** \( \pi \times 0.01 \frac{Nm^2}{C} \)

**B.** \( -\pi \times 0.01 \frac{Nm^2}{C} \)

**C.** \( \pi \times 0.02 \frac{Nm^2}{C} \)

**D.** \( -\pi \times 0.02 \frac{Nm^2}{C} \)
Flux through Closed Surfaces

We integrate over this “Gaussian” surface

The net flux of the electric field through the closed surface:

\[ \Phi_E = \oint \vec{E}(\vec{r}) \cdot d\vec{A} \]

the integral is taken over the whole surface

Convention:
the vector normal to the closed surface points \textbf{outward}.
Example: Point Charge at the Center of a Spherical Surface

At each point of the surface $\vec{E} \parallel d\vec{A}$ (cosφ=1)

$$\Phi_E = \oint \vec{E}(\vec{r}) \cdot d\vec{A} = E(r)4\pi r^2$$

$$= \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$$ - doesn’t depend on r

- holds for any vector field whose strength $\propto \frac{1}{r^2}$. (electrostatic, gravitational)

The flux of the gravitational field of Earth through Earth’s surface?

$$\Phi_g = -g \cdot 4\pi R^2 = -G \frac{M}{R^2} 4\pi R^2 = -4\pi GM$$
What happens if we place the charge *outside*?

\[ \Phi_E = 0 \]

consequence of \(1/r^2\) field dependence = continuity of field lines

The flux magnitudes *are the same* for these two surfaces.
Example: gravitational field flux

\[ \Phi_g = -4\pi Gm \]

Gaussian surface

mass \( m \)

Does the result of flux calculation depends on

1. the shape of the G. surface?

2. The position of the mass within the surface?

3. Presence of the Earth nearby?
Two point charges, \(+q\) (in red) and \(-q\) (in blue), are arranged as shown.

Through which closed surface(s) is the net electric flux equal to zero?

A. surface A  
B. surface B  
C. surface C  
D. surface D  
E. both surface C and surface D
Gauss’ Law

The total flux of the electric field though any closed surface is proportional to the net charge inside the surface.

\[ \Phi_E \equiv \oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \sum q_i \]

The net charge:

\[ Q = \sum q_i \quad Q = \int_{\text{volume}} \rho(r) \, d\tau \]

charges
the same fluxes for all three surfaces

charge density
element of volume

volume
Gauss’ Law (cont’d)

➢ **Electrostatics:** Gauss’ Law $\equiv$ Coulomb’s Law.

➢ **Electrodynamics:** Gauss’ Law $\checkmark$ Coulomb’s Law

(the total flux doesn’t depend on the position of charges, so they can move inside the closed surface!)

Gauss’ Law doesn’t imply action-at-a-distance, and we don’t need to modify it in electrodynamics. In fact, this is the first Maxwell’s equation.
Applications of Gauss Law

\[ \int \vec{E}(\vec{r}) \cdot d\vec{A} = \frac{Q_{\text{net}}}{\varepsilon_0} \]

Gauss’ Law (GL) in its integral form (one scalar equation) is not sufficient for finding three components of a vector field \( \vec{E} \).

However, GL is very useful whenever it is possible to reduce a 3D (vector) problem to a 1D (scalar) problem. The key is the proper symmetry of a problem.

«Useful» symmetries:

- spherical
- cylindrical + translational
- plane

An intelligent choice of a Gaussian surface is crucial!

Example of an «insufficient» symmetry (cylindrical but without translational): an electric dipole.
**Example:** electric field of a uniformly charged ball. Radius \( R \), total charge \( Q \).

Gaussian surface: a sphere centered at \((\cdot)\) \( O \). Its radius \( r \) can be smaller or larger than \( R \).

\[
\rho = \frac{Q}{\frac{4}{3}\pi R^3}
\]

charge density

\[
\Phi_E = \frac{Q}{4\pi \varepsilon_0 R^2}
\]

flux

\[
E(r) \cdot 4\pi r^2 = \frac{q(r' \leq r)}{\varepsilon_0} = \begin{cases} 
\frac{4\pi r^3}{3\varepsilon_0} = \frac{Q r^3}{\varepsilon_0 R^3} & (r < R) \\
\frac{Q}{\varepsilon_0} & (r \geq R)
\end{cases}
\]

enclosed charge

\[
\vec{E}(r) = E(r)\hat{r}
\]

\[
E(r) = \begin{cases} 
\frac{Qr}{4\pi \varepsilon_0 R^3} & (r < R) \\
\frac{Q}{4\pi \varepsilon_0 r^2} & (r \geq R)
\end{cases}
\]

The field outside the sphere is the same as that for a point charge \( Q \) at the sphere’s center.
**Plane Symmetry**

**Example:** the field of uniformly charged infinite plane.

Gaussian surface: a rectangular box (or cylinder) centered at the plane.

\[
E(x) \cdot 2A = \frac{\sigma A}{\varepsilon_0}
\]

\[
\text{flux}
\]

\[
E(x) = \frac{\sigma}{2\varepsilon_0}
\]

\[
\vec{E}(x) = \pm E(x)\hat{x}
\]

\[
\sigma - \text{the surface charge density [C/m}^2]\]

\[
\varepsilon_0
\]
Superposition

\[ E_{\text{net}} = E_1 + E_2 \]

\[ E_{\text{net}} = E_1 + E_2 \]
Gauss’ Law: works in electrodynamics, in electrostatics it is equivalent to Coulomb’s Law.

Powerful tool for computing the electric fields if a problem is essentially 1D due to symmetry.

Next time: Lecture 4. Applications of Gauss’ Law, Conductors in Electrostatics. §§ 22.5

Read about Metals and Dielectrics.
Appendix I. Cylindrical Symmetry + Translational Symmetry along the axis of symmetry

Example: the field of uniformly charged infinitely long wire.

Gaussian surface: a cylinder of length $L$ centered at the wire.

$E_\perp = E$

$E_\perp = 0$

$r$ - the linear density of charges (the unit length charge)

$E(r) \cdot 2\pi rl = \frac{\rho l}{\varepsilon_0}$

$E(r) = \frac{\rho}{2\pi \varepsilon_0 r}$

$E(r) = E(r)\hat{r}$

$E(r)$

$\propto \frac{1}{r}$
Example 1: A charge $q$ is at the center of a cube. What’s the flux of $E$ through the shaded side?

$$\Phi_E = \frac{q}{\epsilon_0} \times \frac{1}{6}$$

Example 2: A charge $q$ is at the back corner of a cube. What’s the flux of $E$ through the shaded side?

$$\Phi_E = \frac{q}{\epsilon_0} \times \frac{1}{24}$$